

RECREATIONS
FOR
GENTLEMEN *and* LADIES:
BEING INGENIOUS
SPORTS
AND
PASTIMES:

CONTAINING

Many curious Inventions, pleasant Tricks on
Cards and Dice; Arithmetical Sports; new
Games; Rules for assuredly winning at all
Games, whether of Cards or Dice; Recre-
ative Fire-works; Tricks to promote Diver-
sion in Company, and other Curiosities.

Translated from the FRENCH, of
MONSIEUR OZANAM.

The FOURTH EDITION.

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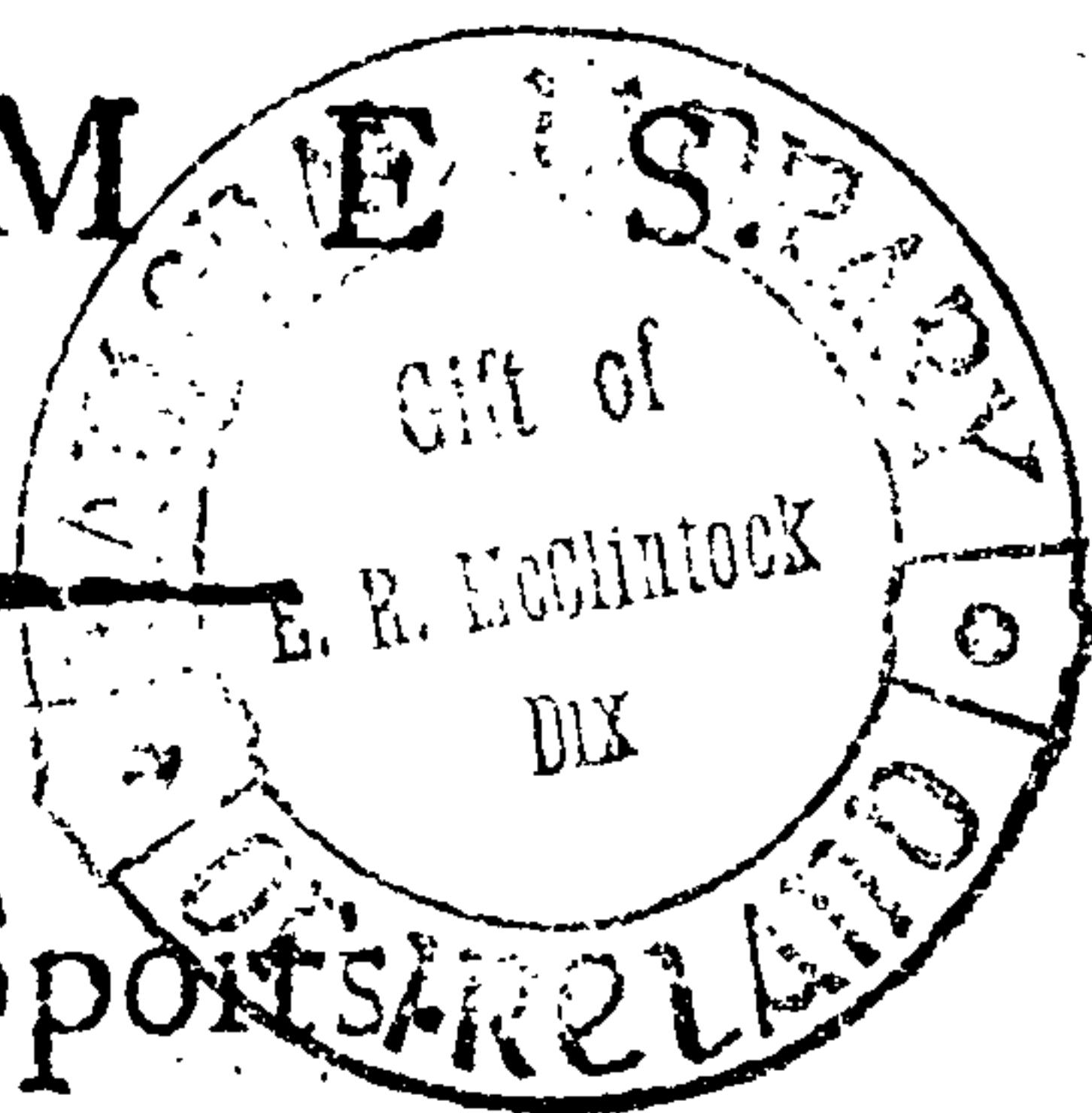
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S P O R T S

A N D

P A S T I M E



Arithmetical Sports

S P O R T I.

Three jealous Husbands with their wives being ready to pass by night over a river, do find at the river side a boat which can carry but two persons at once, and for want of a waterman they are necessitated to row themselves over the river at several times: The question is, how those six persons shall pass 2 by 2, so that none of the 3 wives may be found in the company of 1 or 2 men, unless her husband be present.

THEY must pass in this manner, viz. first two women pass, then one of them brings back the boat, and repas-

ses with the third woman; that done, one of the three women brings back the boat, and sitting down upon the ground with her husband, permits the other two men to pass over to find their wives: then one of the said men with his wife brings back the boat, and placing her upon the ground, he takes the other man, and repasses with him: Lastly, the woman who is found with the three men enters the boat, and at twice goes to fetch over the other two women.

S P O R T I I.

A Countryman having a fox, a goose, and a peck of corn, in his journey came to a river, where it so happened that he could carry but one over at a time. Now, as no two were to be together that might destroy each other; so he was at his wit's end how to dispose of them; for, says he, though the corn can't eat the goose, nor the goose eat the fox, yet the fox can eat the goose, and the goose eat the corn: The question is, how he must carry them over.

FIRST he must carry over the goose, leaving the fox and corn, (for the fox will not eat the corn) then, returning back, he may carry over the fox, bringing the goose back again; then leaving the goose, he may carry over the corn; lastly, he must return to fetch the goose.

SPORT

S P O R T III.

Two merry companions are to have equal shares of eight gallons of wine, which are in a vessel containing exactly eight gallons: now to make this equal partition, they have only two other empty vessels, of which one contains five gallons, and the other three; the question is, how they shall exactly divide the wine by the help of those three vessels?

TO answer this question; let us call the vessel of 8 gallons A, the 5 gallon vessel B, and the three gallon vessel C. We suppose there are 8 gallons of wine in the vessel A, and the other two, B and C,

are empty, as you see in D.

Having filled the vessel B

with wine out of the vessel A, D

in which there will then re- E

main but 3 gallons, as you see F

at E; fill the vessel C with wine G

out of the vessel B, in which, H

by consequence, there will I

then remain but 2 gallons, as K

you see at F. This done, pour

the wine of the vessel C into the vessel A,

where there will then be 6 gallons, as you see

in G; and pour the 2 gallons of the vessel B

into the vessel C, which will then have 2 gal-

lons, as you see at H; then fill the vessel B

with wine out of the vessel A, by which means

there will remain but 1 gallon in it, as you see

A 3

at

	8	5	3
	A	B	C
D	8	0	0
E	3	5	0
F	3	2	3
G	6	2	0
H	6	0	2
I	1	5	2
K	1	4	3

at I; and conclude the operation by filling the vessel C with wine out of the vessel B, in which there will then remain just 4 gallons, as you see at K; and so the question is solved.

If, instead of the vessel B, you would have the 4 gallons to remain in A, which we supposed to be filled with 8 gallons; fill the vessel C with wine, out of the vessel

A, and so there will remain

but 5 gallons in it, as you see

at D; pour the 3 gallons of

the vessel C into the vessel B.

which will then have 3 gal-

lons of wine, as you see at E;

and having again filled the

vessel C with wine out of the

vessel A, where there will then

remain but 2 gallons, as you

see at F; fill up the vessel B

with wine out of C, where there will then re-

main but 1 gallon, as you see at G; at last,

having poured the wine of the vessel B into the

vessel A, where there will then be 7 gallons, as

you see at H; pour the gallon of wine that is

in C into the vessel B, which by consequence

will have only 1 gallon, as you see at I; fill the

vessel C with wine out of the vessel A, where

there will then remain just 4 gallons, pursuant

to the demand of the question, as you see at K.

	8	5	3
A	B	C	
8	0	0	
D	5	0	3
E	5	3	0
F	2	3	3
G	2	5	1
H	7	0	1
I	7	0	1
K	4	1	3

SPORT

S P O R T IV.

A blind Abbess, visiting her Nuns, who were equally distributed in eight cells built at the four corners of a square, and in the middle of each side; finds an equal number of persons in each row or side containing three cells: At a second visit, she finds the same number of persons in each row, though their number was enlarged by the accession of four men: And coming a third time, she still finds the same number of persons in each row, though the four men were then gone, and had carried each of them a nun with them.

TO resolve the first case, when the 4 men were got into the cells, we must conceive it so, that there was a man in each corner cell, and that two nuns removed from thence to each of the middle cells: At this rate, each corner

3	3	3
3	24	3
3	3	3

cell contained one person less than before, and each middle cell 2 more than before; suppose then that at the first visitation, each cell contained three nuns, and so that there were 9 in each row, and 24 in all; at the second visit, which is the first case in question, there must

have been 5 nuns in each middle cell, and 2 persons, viz. 1 man and 1 nun in each corner cell; which still makes 9 persons in each row.

A 4

To

2	5	2
5		5
2	5	2

4	1	4
1		1
4	1	4

To account for the second case, when the 4 men were gone, and 4 nuns with them; each corner cell must have contained 1 nun more than at the first visit, and each middle cell 2 fewer: And thus according to the supposition laid down, each corner cell contained 4 nuns, and there was only 1 in each middle cell; which still make 9 in a row, though the whole number was but 20.

Note, Though at first sight it may be thought by some that the four foregoing Sports cannot be resolved by any certain rule, but only by many trials; yet by infallible argumentation and discourse, the solution of those questions may be found out, or else the impossibility of them, if by chance they should be propounded impossible.

S P O R T V.

Fifteen Christians and fifteen Turks being at sea, in one and the same ship in a terrible storm, and the pilot declaring the necessity of casting the one half of those persons into the sea, that the rest might be saved; they all agreed, that the persons to be cast away should be set out by lot after this manner, viz. the thirty persons should be placed in a round form like a ring,

ring, and then beginning to count at one of the passengers, and proceeding circularly, every ninth person should be cast into the sea, until of the thirty persons there remained only fifteen. The question is, how those thirty persons ought to be placed, that the lot might infallibly fall upon the fifteen Turks, and not upon any of the fifteen Christians?

FOR the more easy remembering of the rule to resolve this question, I shall pre-suppose the five vowels, *a, e, i, o, u*, to signify five numbers, to wit, (*a*) one, (*e*) two, (*i*) three, (*o*) four, (*u*) five; then will the rule itself be briefly comprehended in these two following verses:

*From Numbers, Aid, and Art,
Never will fame depart.*

In which verses you are principally to observe the vowels, with their correspondent numbers before assigned; and then beginning with the Christians, the vowel *o* [in *from*] signifies that four Christians are to be placed together; next unto them the vowel *u* [in *Num.*] imports that five Turks are to be placed together; in like manner *e* [in *bers*] denotes two Christians; *a* [in *Aid*] one Turk; *i* [in *Aid*] three Christians; *a* [in *and*] one Turk; *a* [in *Art*] one Christian; *e* [in *ne*] two Turks; *e* [in *ver*] two Christians; *i* [in *will*] three Turks; *a* [in *Fame*]

Fame] one Christian; *e* [in *Fame*] two Turks; *e* [in *de*] two Christians; *a* [in *part*] one Turk.

The invention of the said rule, and such like, depends upon the subsequent process, *viz.* If the number of persons be thirty, let thirty figures or cyphers be placed circularly, or else in a right line as you see,

oo

That done, begin to count from the first, and mark the ninth, (or what other shall be assigned) by putting a point or cross over it; then count forward from that which you have marked, and place another point over the next ninth; and continue to do the same, beginning again when you shall be at the end (if the cyphers are placed in a right line) and passing over those which you had already marked, until you have marked the number required, as in the example propounded, until you have marked fifteen; for then all the cyphers marked shall be those which must be cast away, and the others those that are to remain. Hence it is evident, that if you observe how those cyphers marked, are disposed among those which are not marked, you will easily make a rule for any number whatsoever.

By this invention (as some conjecture) the famous historian, *Josephus* the Jew, preserved his life very subtilly in the cave, to which himself and forty of his countrymen had fled from the furious and conquering *Romans* at the siege
of

of *Jotapata*: For his said countrymen having most wickedly resolved to kill one another, rather than yield to their enemies; he at length (when no arguments that he could use, would dissuade them from so horrid an act) prevailed with them to execute their tragical design by lot; and so by the help of the aforesaid artifice, as we may suppose, himself, with one other person only remained alive, after the rest were inhumanly murdered, they agreed to put an end to the lot, and thereby save their lives. This story you may see at large in the fourteenth chapter of the third book of the history of *Josephus*, of the wars of the Jews. /

S P O R T VI.

To subtract, with one single operation, several sums, from several other sums given.

TO subtract all the sums that are under the line at B, from all the sums above the line at A; begin by adding the numbers of figures of the right-hand column under the line, saying, 8 and 4 is 12, and 2 makes 14; which taken from the nearest tens, *viz.* 20, there remains 6; which we add to the corresponding column above, saying 6 and 8 make 14, and 2 is 16, and 4 make 20, and 3 make 23: here we write 3 underneath; and in regard there are just two tens, as before, we retain or carry nothing. This done, we add after the same manner, the numbers

56243	
84564	
3252	A
26848	
2942	
3654	B
2308	
162003	

numbers of the next lower column, saying, 0 and 5 is 5, and 4 make 9; which taken from the nearest ten, leaves 1; which we add, as above, to the superior correspondent column, saying, 1 and 4 make 5, and 5 make 10, and 6 make 16, and 4 make 20: here we set 0 underneath, and there being here two tens, whereas in the inferior corresponding column there was but one, we keep or carry the difference 1 to be taken from the next inferior column, because we found more tens in A than in B: For had we found fewer in A than in B, we must have added the difference: and if it should so fall out, that this difference can not be taken from the inferior column, for want of significant figures, as it happens here in the fifth column; we must add it to the superior column, and write the whole sum under the line. Thus in the example proposed, we have 162003, for the remainder of the subtraction.

S P O R T VII.

Compendious ways of MULTIPLICATION.

TO multiply any number, 128 for instance by a number that's the product of the multiplication of two other numbers; 24 for instance, the product of the multiplication of 4 and 6, or of 3 and 8: we multiply the number proposed 128 by 4, and the product 512 by 6, (or else 128 by 3, and the product by 8) and have 3072 for the required multiplication.

Hence

Hence it follows, that to multiply a number proposed by a square number, we must multiply the number proposed by the side or the root of the square; and then the product by the same side again. Thus to multiply 128 by 25, we multiply it by 5, and the product by 5 again.

To multiply any number, 128 for instance, by a number that's the product of the multiplication of *three* other numbers, as 108 the product of 2, 6, and 9, or of 3, 6, and 6: we multiply 128 by 2, the product by 6, and the second product by 9; or else 128 by 3, the product by 6, and the second product by 6.

The consequence of this is, that to multiply any number proposed, by a cube-number, we must multiply it first by the side or root of the cube; then the product of that multiplication by the same root, and the second product by the side again. As, to multiply 128 by 125, the cube-root of which is 5, we multiply 128 by 5, and the product 640 by 5 again, and the second product 3200 by 5 again. Thus to find how many cubical feet are in 32 cubical toises, we multiply 32 by 6, the product of that by 6, and the second product by 6.

To multiply any number by what power you will of five, add to the number proposed, on the right-hand, as many cyphers as the exponent of the power contains unites, as, one cypher for 5, two for its square 25, three for its cube 125, and so on: and divide the numbers thus augmented by the like power from 2, that is, 2 for

5, 4 for its square 25, 8 for its cube 125, and so on.

Thus to multiply 128 by 5, we divide 1280 by 2, and the quotient 640 is the product of the multiplication: but to multiply 128 by 25 the square of 5, we divide 12800 by 4 the square of 2, and the quotient is the product demanded; and to multiply the same number 128 by 125 the cube of 5, we divide 128000 by 8 the cube of 2. And so on.

To know how many inches are in 53 foot, we multiply 53 by 12; or it might be done by multiplying 53 by 2, and the product 53 by 6; or 53 by 3, and the product by 53 4. But there's a way of doing it without any multiplication; *viz.* by setting
 ——— down 53 under 53, and then 53 again
 636 under both, advancing it a column to the left, so as to make 3 stand under 5; for the sum of these three is 636, the number of inches contained in 53 foot, or of pence in 53 shillings.

To multiply together two numbers composed of several figures, 12, for instance, and 18; we reduce the first number 12, into these three parts, each of which consists only of one figure, 2, 4, and 6; and in like manner the second number, 18, into 4, 6, 8; each of which last must be multiplied by 2, the first part of the first number; and then by 4, the second figure of the same first number; and at last by 6, the third part: and the sum of all these products answers the demand.

SPORT

S P O R T VIII.

Division shortened.

TO divide a large number by a smaller, by only addition and subtraction, as 1492992 by 432; we commonly put the divisor to the left, under 1492, to know how many times 'tis contained in that number. But yet we may save ourselves that labour by making a tariff of the divisor; for which end we place it on the right over against 1; then add to itself, or double it, and place that over against 2: Then we add it to the double, and place the sum opposite to 3; adding it to the triple, we have its quadruple opposite to 4; as the additional of itself to the quadruple, gives the quintuple opposite to 5; and so of the other multiplies opposite to 6, 7, 8, 9, 10: The last of which, viz. the multiple corresponding to 10, ought, if the table is right done, to be the single divisor with a cypher on the right-hand.

1	432	1492992	(3456
2	864	1296 . .	
3	1296	<hr/>	
4	1728	1969	
5	2160	1728	
6	2592	<hr/>	
7	3024	2419	
8	3456	2160	
9	3888	<hr/>	
10	4320	2592	
		2592	
		<hr/>	
		000	

Having thus prepared your table, proceed in the common way of division; and every time you have occasion to know how often your divisor is contained in the corresponding number, look in your table for the nearest number that does not exceed; and the number to which that is opposite gives you at one view the figure you are to put in your quotient. As, in the beginning of the division here exemplified, you want to know how often 432 is to be found in 1492; in your table, you find 1296 (the nearest number to 1492, and not exceeding it) opposite to 3, and accordingly 3 is the first figure of your quotient; and so of all the rest.

This way is very convenient, when we have occasion to divide large numbers by a smaller number; for the tairiff of our divisor keeps us from being at a stand, by resolving us readily upon all our divisions. This is frequently the case of surveyors of land, who have occasion to divide large numbers by 144, when they want to reduce square inches into square feet: or by 1728, when they want to reduce cubical inches into cubical feet.

To divide any number by what power you will of 5, multiply it by the like power of 2, and cut off from the right hand of the product as many figures as there are unites in the degree of the power; the remaining figures on the left will represent the quotient of the division, and those struck off will be the numerator of a fraction, the denominator of which will be the like power of 10.

To

To divide any number by a smaller, that is the product of the multiplication of two yet smaller numbers, divide the number proposed, by one of the two smaller, and the quotient by the other; and the second quotient arising from the last division, is what you want.

Thus to divide 20736 by 24, the product of 3 and 8, or of 4 and 6, we take the 8th part of its 3d, or the 6th part of its 4th, or, (which is the same thing) we take the third of its 8th part, or the 4th of its 6th, and our quotient proves 1728.

Hence to reduce square feet to square toises, (a toise is six foot) we must take the 6th part of the 6th part of the number proposed of square feet, because a square toise is 36 square feet, and 6 times 6 is 36. Thus to reduce 542 square feet to square toises, we must take the 6th part of $90\frac{2}{3}$ (the 6th part of 542) and so have 15 square toises and 2 square feet, as the value of 542 square feet.

S P O R T IX.

Curious properties of Numbers.

I NUMBER 9 has this property; that when it multiplies any number of integers whatsoever, the sum of the figures in the product is divisible by 9: thus 53, multiplied by 9, makes the product 477; the figures of which, added together, *viz.* 7 and 7 and 4 make 18, which is exactly divisible by 9.

B.

Take

II. Take any two numbers whatsoever, either one of the two, or their sum, or their difference is divisible by 3: thus, of the two numbers 6 and 5, 6 is divisible by 3; of 11 and 5, the difference 6 is divisible by 3; of 7 and 5 the sum 12 is divisible by 3.

III. The product arising from the multiplication of two numbers, the squares of which make a joint square number, is divisible by 6: thus 12 the product of 3 and 4, the squares of which, viz. 9 and 16, make together the square number 25; this 12, I say, is divisible by 6.

To find two numbers, the squares of which make together a square number, multiply any two numbers, the one by the other, and the double of the product will be one of the two numbers demanded, and the difference of their squares will be the other. Thus in 2 and 3, the double of their product 12, and 5 the difference of their squares (4 and 9) are two numbers of that quality, that their squares 144 and 25 make together the square number 169, the root of which is 13. See Sport X and XI.

IV. The sum and the difference of any two numbers, the squares of which differ by a square number, are each of them, either a square number, or the half of one; thus, take the number 6 and 10, their squares 36 and 100 differ by the square number 64; their sum is 16, and their difference 4, each of which is a square number: then take 8 and 10 for the two numbers, their squares 64 and 100, differ by the square number 36; and the sum 18, and the difference

difference 2, are the halves of the two square numbers 36 and 4.—

To find two numbers, the sum and the difference of which, are, each of them, a square number, In which case, the squares of these two numbers will likewise differ by a square number; pitch upon any two numbers, as 2 and 3, the product of their multiplication is 6, their squares are 4 and 9: 13 the sum of the two squares, and 12 the double of the product of their multiplications, are the numbers we look for; for their sum 25, and their difference 1 are both square numbers; and further, their squares 169 and 144 differ by the square number 25.

To find two numbers, the sum and difference of which are each of them the half or the double of a square number; in which case their squares will likewise differ by a square number; take any two numbers, as 2 and 3, the squares of which are 4 and 9; 13 the sum of these two squares, and 5 the difference, are the two numbers demanded; for their sum 18 and their difference 8, are the halves of the two square numbers 36 and 16, and the doubles of the two square numbers 9 and 4; and farther, their squares 169 and 25, differ by the square number 144, the root of which is 12.

V. Every square number ends either with two cyphers, or with one of the 5 figures 1, 4, 5, 6, 9, which serves for a rule, to distinguish when a number proposed is not square, viz. when it does not end as above; nay, if it does end

with two noughts, and these are not preceded by any of the foregoing five figures, we may rest assured it is not square.

VI. Every square fraction, that is, every fraction that has its square root, is such, that the product of the multiplication of the numerator by the denominator is square. Thus *we know a fraction is not square*, when that does not happen. Take the fraction $\frac{28}{63}$, we know it to be square, because 1764, the product of 28, multiplied by 63, is a square number, having 42 for its root: and so the square root of the proposed fraction is $\frac{42}{63}$, retaining the same denominator; or $\frac{28}{42}$, retaining the same numerator, for either of these is equivalent to $\frac{2}{3}$, for the square root of the proposed fraction $\frac{28}{63}$ or $\frac{4}{9}$.

VII. Any cubical fraction, *i. e.* any that has its cube root, is such, that if you multiply the numerator by the square of the denominator, or the denominator by the square of the numerator, the product has its cube root; and it is by this rule that *we know when a fraction is a cube fraction*, such is $\frac{24}{375}$ for 3375000, and 216000, the two products of the two ways of multiplication just mentioned, have 150 and 60 for their cube roots, and so the cube-root of the fraction $\frac{24}{375}$ is $\frac{150}{375}$ retaining the same denominator, or $\frac{24}{60}$ retaining the same numerator, for each of these fractions is equal to $\frac{2}{5}$ as the cube-root of the proposed fraction $\frac{24}{375}$.

VIII. Though it is not possible to find two homogeneous powers, the sum and difference of
of

of which are each of them a power of the same degree, that is, square numbers if the two first are squares, and cube numbers if they are cubical, &c. yet it is possible, and very easy, to find two triangular numbers, the sum and difference of which, are each of them a triangular number.

Thus 15 and 21 are two triangular-numbers, the sides of which are 5 and 6, and their sum 36; and the difference 6 are likewise triangular numbers, having 8 and 3 for their sides. Again, 780 and 990 are triangular numbers, the sides of which are 39 and 44, and their sum 1770, and the difference 210 are likewise triangular numbers, having 59 and 20 for their sides. Once more, 1747515 and 2185095 are triangular numbers, having 1869 and 2090 for their sides; and their sum 3932610 and the difference 437580 are likewise triangular numbers, the sides of which are 2804 and 935.

By a *Triangular Number* we understand the sum of the natural numbers, 1, 2, 3, 4, 5, 6, beginning with unit, and rising to what number you will, the last and the greatest of which is called the side. Thus we know that 10 is a triangular number, the side of which is four, by reason that it is equal to the sum of the first four natural numbers, 1, 2, 3, 4, the last and greatest of which is 4. It is called *Triangular*, because you may dispose 10 points in the form of an equilateral triangle, each side of which contains 4, and hence it was that 4 got

the name of the side of the triangular number 10.

To know if a number proposed is triangular, you must multiply it by 8, and add 1 to the product, for if the sum be square, the proposed number is triangular. Thus we know that 10 is triangular, because 81 (the sum of its multiplication by 8, with the addition of 1) is a square number, having 9 for its root.

IX. The difference of two homogeneous powers, as of two square numbers, of two cube numbers, &c. is divisible by the difference of their sides. Accordingly we find that 21, the difference of the two square numbers 25 and 4, the sides of which are 5 and 2, is divisible by 3, the difference of the sides or roots, the quotient 7 being always equal to the sum of the same sides or roots; and that 117, the difference of the cubes 125 and 8, the roots of which are 5 and 2, is divisible by 3 the difference of the roots, the quotient 39 being equal to the product of the said roots multiplied one into another, *viz.* 10 added to 29, the sum of their squares 25 and 4.

X. The difference of the two homogeneal powers, the common exponent of which is an even number, is divisible by the sum of their roots. Thus, 21 the difference of the two square numbers 25 and 4, the roots of which are 5 and 2, is divisible by 7, the sum of the said roots, the quotient 3 being equal to the difference of the roots, and 609 the difference of the bi-quadrats 625 and 16, the roots of which

which are 5 and 2, is divisible by 7, the sum of the roots, the quotient 87 being equal to the product arising from 3, the difference of the roots, multiplied with 29, the sum of their squares 25 and 4.

XI. The sum of two homogeneous powers, the common exponent of which is an odd number, is divisible by the sum of their roots. Thus we know that 133 the sum of the two cubes 125 and 8, the roots of which are 5 and 2, is divisible by 7 the sum of these roots, the quotient 19 being equal to the excess of the sum of the squares of the roots (29) above the product of the roots (10). And that 3157 the sum of the two sur-solids 3125 and 32, the roots of which are 5 and 2, is divisible by 7 the sum of the roots; the quotient 451 being equal to the excess of 741 the sum of the bi-quadrat powers of the roots 5 and 2 (625, 16) and of the square of the product of the roots (100) its excess, I say, above 290 the product of the sum of the squares of the same roots (29) multiplied by 10 the product of the roots themselves.

XII. All the powers of the natural numbers 1, 2, 3, 4, 5, 6, &c. have as many differences as their exponents contain units, the last differences being always equal among themselves in each power, that is, the second differences, or the differences of the differences, in the squares 1, 4, 9, 16, 25, 36, &c. for these second differences make 2, the first being the uneven numbers 3, 5, 7, 9, 11, &c. The third differences, or the differences of the dif-

ferences of the first differences in the cubes 1, 8, 27, 64, 125, 216, &c. for these third differences make 6, the first being 7, 19, 37, 61, 91, &c. and the second differences, i. e. the differences of these differences being 12, 18, 24, 30, &c. which rise by 6 for the third difference, and so of the rest.

The same thing happens to Pollygon numbers formed by the continual addition of numbers in continual arithmetical progression, which are called Gnomons, and of which the first is always an unite, which is virtually any Polygon number. The same is the case with Pyramidal numbers, which are formed by the continual addition of Polygon numbers considered as Gnomons, the first of which is always unit: and in like manner with the Pyramido Pyramidal numbers, which are produced by the continual addition of Pyramidal numbers, considered as Gnomons, the first of which is always unity.

When the Gnomons rise, or exceed one another by one, as 1, 2, 3, 4, 5, 6, &c. the Polygon numbers 1, 3, 6, 10, 15, 21, &c. which are formed from them are called Triangular, the property of which is such, that each of them being multiplied by 8, and the product enlarged by unity, the sum is a square number, as we intimated above. And farther, 9 the sum of the second and the third, omitting the first, is a square number, and 36 the sum of the fifth and the sixth, omitting the fourth, is likewise square, and so on.

When

When the Gnomons rise, or exceed one another by two unites, as the odd numbers 1, 3, 5, 7, 9, 11, &c. the Polygon numbers formed from them, 1, 4, 9, 16, 25, 36, &c. are square numbers; and when the Gnomons increase by three unites, as 1, 4, 7, 10, 13, 16, &c. the numbers formed from them, 1, 5, 12, 22, 35, 51, &c. are called Pentagons, and have this peculiar quality, that each of them being multiplied by 24, and 1 added to the product, the sum is a square number, by which rule we know when a proposed number is Pentagon, and so of the others.

To find the sum of as many triangular numbers as you will, commencing from unit, of these eight for instance, 1, 3, 6, 10, 15, 21, 28, 36, multiply the given number 8 by the next follower 9, and the product 72, by the next after that 10, and divide the second product 720 by 6, the quotient gives you 120 the sum demanded.

The sum of all these infinite fractions $\frac{1}{3}, \frac{1}{6}, \frac{1}{10}, \frac{1}{15}, \frac{1}{21}, \frac{1}{28}, \frac{1}{36}, \dots$ the common numerator of which is 1, and the denominators of which are triangular numbers, their sum I say, is just 1.

To find the sum of as many square numbers from an unit as you will, of these eight, for example, 1, 4, 9, 16, 25, 36, 49, 64, take 36, the last of as many triangular numbers, viz. 1, 3, 6, 10, 15, 21, 28, 36, from 240 the double of this sum 120, and the remainder 204 is the sum you want.

XIII. The cubes, 1, 8, 27, 64, 125, 216, &c. of the natural numbers, 1, 2, 3, 4, 5, 6, &c. are such, that the first 1 is a square number, the root of which 1 is the first triangular number; the sum of the two first, 1 and 8, viz. 9 is a square number, the root of which 3 is the second triangular number 36, the sum of the three first, 1, 8, 27, is a square number, the root of which 6 is the third triangular number, and so on. And therefore if you want to find the sum of any number of cubic numbers from an unit, of these fix for example, 1, 8, 27, 64, 125, 216,) the square of the fixth triangular number 21, (441) is the sum you desire.

XIV. Among whole numbers, there is only 2 that being added to itself, makes as much as when multiplied by itself, viz. 4, for all other numbers make more by multiplication than by addition.

Though we cannot find two whole numbers the sum of which is equal to the product of their multiplication, yet we can easily find two fractional numbers, and even in a given Ratio, the sum of which is equal to their product, viz. by dividing the sum of the two terms of the given Ratio by each of the two terms; thus, if you give them the Ratio of the two numbers, 2, 3, divide their sum 5 separately by 2 and by 3, and you will have the two numbers $2\frac{1}{2}$, $1\frac{2}{3}$, which make as much when added together as when multiplied together, viz. $4\frac{1}{6}$.

XV. Any number is the half of the sum of two others equally remote, the one in the way of
of

of defect, and the other in excess. For example, 6 is the half of 12, the sum of the two numbers equally remote, 5 and 7, or 4 and 8.

XVI. The number 37 has this property, that being multiplied by any of these numbers, 3, 6, 9, 12, 15, 18, 21, 24, 27, which are in continual arithmetical progression, all the products are composed of one figure thrice repeated.

37	37	37	37	37	37	37	37	37
3	6	9	12	15	18	21	24	27

111 222 333 444 555 666 777 888 999

XVII. The two numbers 5 and 6 are called Spherical, because their powers terminate in these very numbers. The powers of 5, viz. 25, 125, 625, &c. terminate in 5, and in like manner the powers of 6, viz. 36, 216, 1296, &c. end with 6.

5 has that peculiar quality, that when multiplied by an odd number (as 7) its product terminates in 5, (as 35) and when multiplied by an even number (as 8) its product ends in a cypher, (as 40).

The other number, 6, has likewise this singular quality, that it is the first of the numbers which we call *perfect*, as being equal to the sum of their aliquot parts, for 6 is equal to the sum of its aliquot parts 1, 2, 3; 28 is likewise a *perfect* number, in regard it is equal to the sum of its aliquot parts 1, 2, 4, 7, 14: and one may find an infinity of other perfect number,

as



as 496, which is equal to the sum of its aliquot parts 1, 2, 4, 8, 16, 31, 62, 124, 248.

To find all the perfect numbers in order, make use of the powers of 2, viz. 2, 4, 8, 16, 32, &c. and see which of these powers, when an unit is taken from them, makes a prime number, and you will find in 4, 8, 32, &c. that if you subtract 1 from each of them, the remainders 3, 7, 31, &c. are prime numbers, each of which ought to

be multiplied by the half	2	4	8	16	32
of the corresponding		1	1		1
power, that is, 3 by 2,	—	—			—
7 by 4, 31 by 16, &c.		3	7		31
in order to obtain the		2	4		16
perfect numbers 6, 28,	—	—			—
496, &c.		6	28		496

To find all the Aliquot Parts, or all the divisors of a proposed number, of which an unit is always one. If the number be 8128 (for example) which is likewise a perfect number, divide it by the least number that offers, viz. 2, which is easily done, because 8128 is an even number, so the quotient will be 4064, which set down over against 2 for your second divisor, which may still be divided by the first divisor 2, and so its square 4 may likewise be a divisor, which set down under 2, over against the second quotient 2032 for another divisor, which may still be divided by the first divisor 2, and therefore its cube 8, will likewise be a divisor, which you are to write under the square 4, and opposite to the third quotient 1016 for another divisor:

divisor: thus you go on, till you come to the last divisor that cannot be divided by 2, viz. the sixth quotient 127, which being a prime number, that is, a number that can be divided by nothing but an unit, gives us to know that we have traced all the divisors of the number proposed 8128, and here you see the sum of the divisors is equal to the number proposed, and by consequence it is a perfect number.

1	
2	4064
4	2032
8	1016
16	508
32	254
64	127
—	—
127	8001
	127
	—
	8128

By the same method did we find out all the divisors of the other number 2096128 which is likewise perfect, for as you see it is equal to the sum of its aliquot parts. You see likewise that the last quotient 2047 which answers to 1024 the tenth power of the first divisor 2. is also a prime number, for if it could have been divided by any other number beyond 2, as by 3, it behoved us to have multiplied all the power of the first divisor 2 by this new divisor 3, and to have divided the number proposed and all the quotient by this new divisor 3, in order to have their divisors, as you will see in the following example.

1	
2	1048064
4	524032
8	262016
16	131008
32	65504
64	32752
128	16376
256	8188
512	4094
1024	2047
—	—
2047	2094081

XVIII. The number 120 is equal to the half of 240, the sum of its aliquot parts 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60. The number 672 is likewise equal to the half of 1344 the sum of its aliquot parts, as will appear by observing the method above prescribed, which we shall not now repeat. We may find a great many other numbers that have the same quality; nay, some may be found to be the third, or any other part of the sum of their aliquot parts, which we shall not now insist upon.

XIX. The two numbers 220 and 284 are called *Amiable*, because the first 220 is equal to the sum of the aliquot parts of the latter, 1, 2, 4, 71, 142; and reciprocally the latter 284 is equal to the sum of the aliquot parts of the former, 1, 2, 4, 5, 10, 11, 22, 44, 55, 110. These aliquot parts are easily found by what we have said before, especially if we consider that all numbers that end in 5, or in 0, are divisible by 5.

To find all the *Amiable Numbers* in order, make use of the number 2, which is of such a quality, that if you take 1 from its tripple 6, from its sextuple 12, from the octodecuple of its square 72, the remainders are the three prime numbers 5, 11, and 71, of which 5 and 11 being multiplied together, and the product 55 being multiplied by 4 the double of the number 2, this second product 220 will be the first of the two numbers we look for; and to find the other 284, we need only to multiply the third prime

prime number 71 by 4, the same double of 2, that we used before.

To find two other amiable numbers, instead of 2 we make use of two of its powers that possess the same quality, such as its cube 8; for you subtract an unit from its tripple 24, from its sextuple 48, and from 1152 the octodecuple of its square 64, the remainders are the three prime numbers, viz. 23, 47, 1151, of which the two first, 23, 47, ought to be multiplied together, and their product 1081 ought to be multiplied by 16 the double of the cube 8, in order to have 17296 for the first of the two numbers demanded. And for the other amiable number demanded, which is 18416, we must multiply the third prime number 1151 by 16, the same double of the cube 8.

If you still want other amiable numbers, instead of 2, or its cube 8, make use of its square cube 64, for it has the same quality, and will answer as above.

In regard, it is difficult to know whether a number is prime if it be a large number, we shall at the end of this Sport subjoin a table of all the prime numbers between 1 and 10000.

XX. The squares of the two numbers 31, 34, viz. 961, 1156, are such, that the first 961, with its aliquot parts, 1, 31, makes a sum (993) equal to 1, 2, 4, 17, 34, 68, 289, 578, the aliquot parts of the second 1156.

XXI. The two numbers 26, 20, make each of them with their aliquot parts, the same sum; the first 26 with its aliquot parts, 1, 2, 13, makes

makes 42, and the second (20) with its aliquot parts 1, 2, 4, 5, 10, makes likewise 42.

• The same is the case of 488 and 464, each of them with their aliquot parts making 930: of 11 and 6, each of them with their aliquot parts making 12; and in fine of 17 and 10, which with their aliquot parts make 18 a piece.

Nay, we may find three numbers, each of which with its aliquot parts makes the same sum, as 20, 25 and 41, as also 23, 14, 15 and 46, 51, 71.

We may find two square numbers of the same quality, particularly 16 and 25 the squares of 4 and 5; which are the lowest that can be, and by virtue of which we come at as many more as we will of the same quality, *viz.* by multiplying them by some odd square number, that is not divisible by five. For example, if we multiply each of them by the square number 9, we obtain two other square numbers 144 and 225, each of which with its aliquot parts makes just 403.

XXII. 81 the square of 9, with its aliquot parts, 1, 3, 9, 27, makes a square number (121) the root of which is 11, 400 the square of 20, with its aliquot parts, make the square of 31 (961).

XXIII. 666 the sum of these three triangular numbers 15, 21, 630, the sides of which are 5, 6, 35, is likewise a triangular number, the sides of which is 36. The same is the case of these three triangular numbers 210, 780, 1711, and likewise of these 666, 2628, 5886.

XXIV.

XXIV. 49 the square of 7 has this quality, that 8 the sum of its aliquot parts, 1, 7, is the cube of 2, and 343 the cube of the same number 7, does with its aliquot parts 1, 7, 49, make the square number 400, the root of which is 20. I do not here pretend to direct you how to find out others of the same quality, for unless you light upon them by chance, it is very difficult to trace them without Algebra, which I propose not to mention in this performance.

XXV. 9 the square of 3 has this quality, that 4 the sum of its aliquot parts 1, 3, is the square of 2, 2401 the square of 49 has the same quality, for 400, the sum of its aliquot parts 1, 7, 49, 343 is the square of 20.

XXVI. The two numbers 99, 63, have this quality, that (57) the sum of the aliquot parts of the first, 1, 3, 9, 11, 33, surpasses (41) the sum of the aliquot parts of the second, 1, 3, 7, 9, 21, by the square number 16, the root of which is 4. The same is the condition of 325 and 175; for the sum of the aliquot parts of the first exceeds that of the aliquot parts of the other, by the square number 36.

XXVII. The sum of two numbers that differ by unity, is equal to the difference of their squares, and the sum of the squares of their triangular numbers is likewise a triangular number. Thus 5 and 6 make the sum 11 equal to the difference of their squares 25, 36, and their triangular numbers 15, 21, are such, that 666, the sum of their squares 225, 441, is likewise a triangular number, the side of which is 36.

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XXVIII.

XXVIII. The two triangular numbers, 6, 10, of the two numbers 3, 4, the difference of which is likewise an unity, have this quality, that their sum 16, and their difference 4, are square numbers, having 4 and 2 for roots; and 136 the sum of their squares (36, 100) is a triangular number, the side of which 16 is likewise a square number, the root of which is at the same time a square number, having 2 for its side or root.

The same is the quality of the two other triangular numbers, 36, 47 the sides of which, 8, 9, differ only by unity, for their sum 81, and their difference 9, are square numbers, the roots of which are 9 and 3, and 3321 the sum of their squares (1296, 2025) is a triangular number, the side of which is 81, and that has its square root 9, which again is the square of 3.

There are many other triangular numbers of this quality, that may be found out by subtracting and adding any square number to its square, the halves of the remainder and of the sum being the two triangular numbers demanded. For example, if you subtract 8 the square number 16 from and add it to its square 256, half the remainder 240, and half the sum 272, present us with 120, and 136, for the two triangular numbers thought for, the sides of which are 15, 16, the difference consisting still in unity.

These two triangular numbers thus found, have this farther quality, that the greatest of their sides is always a square number, and the difference

difference of their squares is likewise a square number; and withal their sum is a bi quadrate, equal to the square of their difference, and at the same time to the side of the triangular number that composes the sum of their squares.

XXIX. The difference of the squares of the two numbers in a duplicate Ratio, is equal to the sum of their cubes divided by the sum of their two numbers, and that very sum of their cubes is the third of a cube.

Accordingly, 4 and 8 being in a duplicate Ratio, the difference 48 of their squares, 16, 64, is equal to the quotient resulting from the division of 576 (the sum of their cubes, 64, 512) by 12 the sum of the two numbers, and the very sum of their cubes 576 is the third part of the cube 1728, the root of which 12 is always equal to the sum of the two numbers.

I should never have done, if I pretended here to fetch in all the properties of numbers, which indeed are infinite, and upon that consideration I shall now conclude this Sport with the table of the prime numbers that I promised above.

Table of the Prime Numbers between 1 and 10000.

2	149	337	547	757	997	1223	1471	1699
3	151	347	557	761	—	1229	1481	—
5	157	349	563	769	1009	1231	1483	1709
7	163	353	569	773	1013	1237	1487	1721
11	167	359	571	787	1019	1249	1489	1723
13	173	367	577	797	1021	1259	1493	1733
17	179	373	587	—	1031	1277	1499	1741
19	181	379	593	811	1033	1279	—	1747
23	191	383	599	821	1039	1283	1511	1753
29	193	389	—	823	1049	1289	1523	1759
31	197	397	601	827	1051	1291	1531	1777
37	199	—	607	829	1061	1297	1543	1783
41	—	401	613	839	1063	—	1549	1787
43	211	409	617	853	1069	1301	1553	1789
47	223	419	619	857	1087	1303	1559	—
53	227	421	631	859	1091	1307	1561	1801
59	229	431	641	863	1093	1319	1571	1811
61	233	433	643	877	1097	1321	1579	1823
67	239	439	647	881	—	1327	1583	1831
71	241	443	653	883	1103	1361	1597	1847
73	251	449	659	887	1109	1367	—	1861
79	257	457	661	—	1117	1373	1601	1867
83	263	461	673	907	1123	1381	1607	1871
89	269	463	677	911	1129	1399	1609	1873
97	271	467	683	919	1151	—	1613	1877
—	277	479	691	929	1153	1409	1619	1879
101	281	487	—	937	1163	1423	1621	1889
103	283	491	701	941	1171	1427	1627	—
107	293	499	709	947	1181	1429	1637	1901
109	—	—	719	953	1187	1433	1657	1907
113	307	509	727	961	1193	1439	1663	1913
127	311	509	733	971	—	1447	1667	1931
131	313	521	739	977	1201	1451	1669	1933
137	317	523	743	983	1213	1453	1693	1949
139	331	541	751	991	1217	1459	1697	1951

1973	2237	2477	2749	3037	3293	3607	3889
1979	2239	—	2753	3041	3313	3613	—
1987	2243	2503	2767	3049	3433	3617	3907
1993	2251	2521	2777	3061	3473	3623	3911
1997	2267	2531	2789	3067	3593	3631	3917
1999	2269	2539	2791	3079	3613	3637	3919
—	2274	2543	2797	3083	3371	3643	3923
2003	2281	2549	—	3089	3373	3659	3929
2011	2287	2551	2801	—	3389	3671	3931
2017	2293	2557	2803	3109	3391	3673	3943
2027	2297	2579	2819	3119	—	3677	3947
2029	—	2591	2833	3121	3407	3691	3967
2039	2309	2593	2837	3137	3413	3697	3989
2053	2311	—	2843	3163	3443	—	—
2063	2333	2609	2851	3167	3449	3701	4001
2069	2339	2617	2857	3169	3457	3709	4003
2081	2341	2621	2861	3181	3461	3719	4007
2083	2347	2633	2879	3187	3463	3727	4013
2087	2351	2647	2887	3191	3467	3733	4019
2089	2357	2657	2897	—	3469	3739	4021
209	2371	2659	—	3203	3491	3761	4027
—	2377	2663	2903	3209	3499	3767	4049
2111	2381	2671	2909	3217	—	3769	4051
2113	2383	2677	2917	3221	3511	3779	4057
2129	2389	2683	2927	3229	3517	3793	4073
2131	2393	2687	2939	3251	3527	3797	4079
2137	2399	2689	2953	3253	3529	—	4091
2141	—	2693	2957	3257	3533	3803	4093
2143	2411	2699	2963	3259	3539	3821	4069
2153	2417	—	2969	3271	3541	3823	—
216	2423	2707	2971	3299	3547	3833	4111
2179	2437	2711	2999	—	3557	3847	4127
—	2441	2713	—	3301	3559	3851	4129
2203	2447	2719	3001	3307	3571	3853	4133
2207	2459	2729	3011	3313	3581	3863	4139
2213	2467	2731	3019	3319	3583	3877	4153
2221	2473	2741	3023	3323	3593	3881	4157

4159	4463	4759	5039	5381	5651	5903	6229
4177	4481	4783	5051	5385	5653	5923	6247
—	4483	4787	5059	5393	5657	5927	6257
4201	4493	4789	5077	5399	5659	5939	6261
4211	—	4793	5081	—	5669	5953	6269
4217	4507	4799	5087	5407	5683	5981	6271
4219	4513	—	5099	5413	5689	5987	6277
4229	4517	4801	—	5417	5693	—	6287
4231	4519	4813	5101	5419	—	6007	6299
4241	4523	4817	5107	5431	5701	6011	—
4243	4547	4831	5113	5437	5711	6029	6301
4253	4549	4861	5119	5441	5717	6037	6307
4259	4561	4871	5147	5443	5737	6042	6313
4261	4567	4877	5153	5449	5741	6047	6323
4271	4583	4889	5167	5471	5743	6053	6329
4273	4591	—	5171	5477	5749	6067	6337
4283	4597	4909	5179	5479	5779	6073	6343
4289	—	4909	5189	5483	5783	6079	6353
4297	4603	4911	5197	—	5791	6089	6355
—	4621	4933	—	5501	—	6091	6359
4327	4637	4933	5209	5503	5801	—	6361
4337	4639	4937	5227	5507	5807	6101	6367
4339	4643	4943	5231	5519	5813	6113	6373
4349	4649	4951	5233	5521	5821	6121	6379
4357	4651	4957	5237	5527	5827	6131	6389
4363	4657	4967	5261	5531	5839	6133	6397
4373	4663	4969	5273	5557	5843	6143	—
4391	4673	4973	5279	5563	5849	6151	6421
4397	4679	4987	5281	5569	5851	6163	6427
—	4691	4993	5297	5573	5857	6173	6449
4409	—	4999	—	5581	5861	6197	6451
4421	4703	—	5303	5591	5867	6199	6469
4423	4721	5003	5309	—	5869	—	6473
4441	4723	5009	5323	5623	5879	6203	6481
4447	4729	5011	5333	5639	5881	6211	6491
4451	4733	5021	5347	5641	5897	6217	—
4457	4751	5023	5351	5647	—	6221	6521

6529	6827	7121	7477	7723	8069	8369	8689
6547	6829	7127	7481	7727	8081	8377	8693
6551	6833	7129	7487	7741	8087	8387	8699
6555	6841	7151	7489	7753	8089	8389	—
6563	6857	7159	7499	7757	8093	—	8707
6569	6863	7177	—	7759	—	8419	8713
6571	6869	7187	7507	7789	8101	8423	8719
6577	6871	7193	7517	7791	8111	8429	8731
6581	6883	—	7523	—	8117	8431	8737
6599	6899	7207	7529	7817	8123	8443	8741
—	—	7211	7537	7823	8147	8447	8747
6607	6907	7213	7541	7829	8161	8461	8753
6619	6911	7219	7547	7841	8167	8467	8761
6637	6917	7229	7549	7853	8171	—	8779
6653	6947	7237	7559	7867	8179	8501	8783
6659	6949	7253	7561	7873	8191	8513	—
6661	6959	7243	7573	7877	—	8521	8803
6673	6961	7247	7577	7879	8209	8527	8807
6679	6967	7283	7583	7885	8219	8537	8819
6689	6971	7297	7589	—	8221	8539	8821
6691	6977	—	7591	7901	8231	8543	8831
—	6983	7307	—	7907	8233	8563	8837
6701	6991	7309	7603	7917	8237	8573	8839
6703	6997	7321	7607	7927	8243	8581	8849
6709	—	7331	7621	7933	8263	8597	8861
6719	7001	7333	7639	7937	8269	8599	8863
6733	7013	7349	7643	7949	8273	—	8867
6737	7019	7351	7649	7951	8287	8609	8887
6761	7027	7369	7669	7963	8291	8623	8893
6763	7039	7393	7673	7993	8293	8627	—
6779	7043	—	7681	—	8297	8629	8923
6781	7057	7411	7687	8009	—	8641	8929
6791	7069	7417	7691	8011	8311	8647	8933
6793	7079	7433	7699	8017	8317	8663	8941
—	—	7451	—	8039	8329	8669	8951
6803	7103	7457	7703	8053	8353	8677	8963
6823	7109	7459	7717	8059	8363	8681	8969

8971	9109	9239	9377	9479	9629	9767	9871
8999	9127	9241	9393	9491	9631	9769	9883
—	9133	9257	9397	9497	9643	9781	9887
9001	9137	9277	—	—	9649	9787	—
9007	9151	9281	9403	9511	9661	9791	9901
9011	9157	9283	9413	9521	9677	9793	9907
9013	9161	9293	9419	9533	9679	—	9923
9029	9173	—	9421	9539	9689	9803	9929
9041	9181	9311	9431	9547	9697	9811	9931
9043	9187	9319	9433	9551	—	9817	9941
9049	9199	9322	9437	9587	9719	9829	9949
9059	—	9337	9439	—	9721	9833	9967
9067	9203	9341	9461	9601	9733	9839	9973
9091	9209	9343	9463	9613	9739	9851	—
—	9221	9349	9467	9619	9743	9857	—
9103	9227	9371	9473	9623	9749	9859	—

S P O R T X I.

Of Right-angled Triangles in numbers.

BY a Rectangular Triangle in numbers, we mean three unequal numbers, the greatest of which is such, that its square is equal to the square of the other two, such are 3, 4, 5, for 25 the square of 5 the greatest, which we call the Hypothenuse, is equal to the sum of 9 and 16, the squares of the other two numbers, 3, 4, which we call the sides, taking one for the Base of the right-angled triangle, and the other for the Altitude or Height. Half the product of the Base, and the Altitude, is called the Area, and is always divisible by 3. The reader will observe all along that by the product of two numbers,

numbers, we understand the number arising from their mutual multiplication.

There is an infinite number of right-angled triangles of divers sorts, both in whole and in broken or fractional numbers, but we generally conceive them in integers, among which the first and the least of all is that now mentioned, 3, 4, 5, which has an infinity of fine properties, but it would be tedious to enumerate them, and therefore I shall content myself with observing, that the sum (216) of the cubes (27, 64, 125) of the two sides 3, 4, and of the Hypothenuse (5) is a cube, the root or side of which (6) is equal to its Area.

To find in numbers as many Right-angled Triangles as you will: Take any two numbers, for example 2 and 3, which we call Generating Numbers, multiply them the one by the other, and (12) the double of their product (6) is the side of a right-lined triangle, the other side being equal to (5) the difference of the squares (4, 9) of the generating numbers 2, 3, and the Hypothenuse being equal to (13) the sum of the same squares, 4, 9. And thus you have this right-angled triangle 5, 12, 13, for 169, the square of the Hypothenuse 13 is equal to the sum of 25, 144, the squares of the two sides 5, 12.

The first right-angled triangle, having 1, 2, for its generating numbers is such, that the difference of the two sides 3, 4, is 1; and if you want to find another of the same quality, take 2 the greatest of these generating numbers for the least

least of the two in the triangle demanded; and in order to find the greatest for this second triangle, add 1, the least of the first, to 4 the double of the greatest of the first, and so you have 5 for your greatest generating number of the second right-angled triangle, which consequently is 20, 21, 29, where the difference of the two sides 20, 21, is again 1.

If you desire a third right-angled triangle of the same quality, make use of the last 20, 21, 29, after the same manner as you did the first, taking its greatest generating number for the least of the third, and adding its least to the double of the greatest, for the greatest of this your third triangle; and so observing the same method, you may find a fourth, fifth, &c. as appears by this table.

Sides.	Hypoth.	Generat. numb.
3	4	5
20	21	29
119	120	169
696	697	1025
4059	4060	5741
23660	23661	33461

The first right angled triangle 3, 4, 5, has likewise this quality, that the excess of the Hypothenuse 5 above the side 4, is also 1, forasmuch as the difference of the two generating numbers is 1, and for this reason you may find an infinite number of other right-angled triangles of this quality, if for their generating you

you take two that differ only by unity, as you see in this table.

Bases.	Altitude.	Hypoth.	Generat. numb.
3	4	5	1 2
5	12	13	2 3
7	24	25	3 4
9	40	41	4 5
11	60	61	5 6
14	84	85	6 7

Here you see the first differences of the bases, 3, 5, 7, 9, &c. are equal, and the second differences of the altitudes, 4, 12, 24, 40, &c. are likewise equal; and the same is the case of the Hypothenufes, 5, 13, 25, &c.

Here the Bases are odd numbers, and if you would have them the squares of these odd numbers, only take the altitudes and Hypothenufes for the generating numbers of the triangles you propose, which by consequence will run thus:

Bases.	Heights.	Hypoth.	Gen. numb.
9	40	41	4 5
25	312	313	12 13
49	1200	1201	24 25
81	3280	3281	40 41
121	7320	7321	60 61
169	14280	14281	84 85

If instead of one side you would have the Hypothenufe to be the square number, then your generating numbers must be the sides of a right-

right-angled triangle, as in the following scheme, where you see the Hypothenuse is the square of the greatest generating number, with the addition of 1.

Sides.	Hypoth.	Gen.	numb.
7 24	25	3	4
119 120	169	5	12
336 527	625	7	24
720 1519	1681	9	40
1320 3479	3721	11	60
2184 6887	7225	13	84

The right-angled triangle, 21, 28, 35, has this quality, that the two sides 21, 28, are triangular numbers, the sides of which, 6 and 7, differ only by unity, and the square (1225) of the Hypothenuse (35) is likewise a triangular number, the side of which is 49.

The same is the quality of the triangle 820, 861, 1189, as also of the triangle 28441, 28680, 40391, and of others.

The following right-angled triangles, which may be continued *in infinitum*, are such, that their bases and hypotenuses are triangular numbers, and their heights cubic numbers.

Bases.	Heights.	Hypoth.	Gen.	numb.
6	8	10	1	3
36	27	43	3	6
120	64	136	6	10
300	125	135	10	15
630	216	666	15	21
1176	343	1225	21	28

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You may find as many such triangles as you will, by adding and subtracting a square number from its square, for in the addition half the sum is the hypotenuse, and in subtracting half the remainder is the base, the height being equal to the cube of the root of the first square number: or, which is the same thing, by taking for the generating numbers the triangular numbers in order, as you see in the scheme before us, where the least generating numbers of one right-angled triangle is the greatest of the preceding triangle.

S P O R T XII.

Of Arithmetical Progression.

BY Arithmetical Progression, we mean a series of quantities called Terms, that rise continually by an equal excess, as 1, 3, 5, 7, 9, 11, &c. where the excess is 2; or 1, 4, 7, 10, 13, 16, &c. where the excess is 3; or 2, 6, 10, 14, 18, 22, &c. where they rise by 4 at a time. And so of the rest.

The principal property of arithmetical progression, is this: Take three continual terms, as 6, 10, 14, the sum (20) of the two extremes (6, 14) is equal to the double of the middle term (10). Take four continual terms, as 6, 10, 14, 18, the sum (24) of the two extremes (6, 18) is equal to that of the two middle terms (10, 14). In fine, in a larger number of continual terms, six for instance, as 2, 6, 10,

10, 14, 18, 22, the sum (24) of the two extremes (2, 22) is equal to that of any two terms that lie at an equal distance from them, as 6, 18, and 10, 14. From whence it is easy to conclude, that when a multitude of progressive terms is an odd number, the sum of the extremes, or of those equally remote, is the double of the middle term, as in these five terms, 2, 6, 10, 14, 18; for the sum (20) of the extremes 2, 18, or of the two equally remote, 6, 14, is the double of the middle term, 10.

You may readily find such numbers as have this quality, that the sum of their squares make a square number, or, which is the same thing, the sides of a right-angled triangle in numbers; and that by virtue of this double arithmetical progression, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, where the excess is 2 in fractions, and 1 in whole numbers; for if you reduce the integer with its fraction to a fraction only, as $1\frac{1}{2}$ to $\frac{3}{2}$, the numerator 3 and the denominator 2 will be the sides of the right-angled triangle 3, 4, 5; and in like manner if you reduce $2\frac{1}{2}$ to $\frac{5}{2}$, (which is done by multiplying the whole number 2 by the denominator 2, and adding to the product 10 the numerator 2) the denominator 2, and the numerator 5, will be the sides of the right-angled triangle 2, 5, 13; and so of the rest. Here you may see any odd number may be one of the sides of a right angled triangle in whole numbers.

Instead of the double arithmetical progression, you may make use of this, $1\frac{7}{8}$, $2\frac{11}{8}$, $3\frac{15}{8}$, $4\frac{19}{8}$

$4\frac{1}{2}$, $5\frac{3}{4}$, &c. where the excess is 4 in fractions, and 1 in whole numbers, for if you reduce $1\frac{7}{8}$ to $\frac{15}{8}$, the denominator 8, and the numerator 15, will be the two sides of the right-angled triangle 8, 15, 17; and in like manner if you reduce $2\frac{1}{2}$ to $\frac{5}{2}$, the denominator 12 and the numerator 35 will be the two sides of another triangle 12, 35, 37; and so on. Here you see any odd number may be one of the sides of a right-angled triangle in whole numbers.

In an arithmetical progression, the sum of the terms is equal to the sum of the two extremes, multiplied by half the number of all the terms. And for this reason, in order to find the sum of any number of terms in Arithmetical Progression, for example, the sum of these eight, 3, 5, 7, 9, 11, 13, 15, 17, you must multiply the sum (20) of the two extremes (3, 17) by the number of the multitude of the terms (8) for then half the product (80 the half of 160) is the sum you inquire for.

If on the other hand you know the sum of the terms, the first term itself, and the number or multitude of the terms, you may find out what the terms are, by tracing the excess in this manner. Suppose the given sum of the terms to be 80, the number of them 8, and the first term given 3, divide (160) the double of the sum given (80) by the number given (8) then subtract from 20 the quotient, 6 the double of the first term given 3, and at last divide the remainder 14 by the given number wanting 1, that

that is 7, and the quotient 2 is the excess you look for, which added to the first term gives you 5 for the second, and added to the second 7 for the third, and so on.

If the sum of the terms, their number, and the excess be given, we find out the first term, and by consequence all the rest after the manner of the third question ensuing.

Question I. A gentleman bargains with a bricklayer to have a well sunk upon these terms; he is to allow him three livres for the first toise (a toise is 6 foot) of depth, 5 for the second, 7 for the third, and so on, rising two livres every toise till the well is twenty toises deep. Query, how much will be due to the bricklayer, when he has digged twenty toises deep?

To resolve this question, multiply the 2 livres augmentation money at every toise, by the number of the toises, bating 1, that is by 19, to the product 38 add 6 the double of 3 the number of livres promised for the first toise, then multiply the sum 44 by half the number of all the toises, viz. 10, and the product shews you 440 livres due to the bricklayer for sinking the well 20 toises deep.

Quest. II. A gentleman travelled 100 leagues in eight days, and every day travelled equally further than the preceding day. Now it being discovered that the first day he travelled two leagues, the question is how many days he travelled on each of the other days?

To resolve this question, divide 200, the double of the leagues given 100, by 8 the number
of

of days given, and from the quotient 15, subtract 4, the double of 2 the given number of leagues that he travelled the first day. Divide the remainder 21 by 7, the given number of days wanting one, and the quotient 3 shews that he travelled every day three leagues more than the day before, from whence it is easy to conclude, that since he travelled two leagues the first day, he travelled 5 the second, 8 the third, and so on.

Quest. III. *A traveller went 100 leagues in 8 days, and every day three leagues more than the preceding day. 'Tis asked how many leagues he travelled a day?*

Divide 200, the double of the leagues given 100, by 8 the number of days given, and from the quotient 25 subtract 21, the product of 3 the number of the daily increase multiplied by 7 the given number of days bating 1. The remainder being 4 half it, and that shews you he travelled 2 leagues the first day; from whence it is easy to gather that he travelled 5 the second, 8 the third, and so on.

Quest. IV. *A robber being pursued, travelled 8 leagues a day; an Archer, who was the pursuer, made but 3 leagues the first day, 5 the second, 7 the third, and so on, increasing 2 leagues every day. The question is in how many days the archer will come up with the robber, and how many leagues they will have travelled?*

To resolve this and such like questions, add 2 the number of the daily increase of leagues by the archer, to 16 the double of 8 the num-
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ber

ber of leagues made every day by the robber: from the sum 18 subtract 6 the duplicate of 3, the number of leagues that the archer travelled the first day. The remainder 12, divide by 2 the number of the archer's daily increase; and the quotient 6 will shew you, that the archer will come up with the robber at the end of 6 days, and consequently both of them must by that time have travelled 48 leagues, for 6 times 8 is 48, and the same is the sum of these six terms of Arithmetical Progression, 3, 5, 7, 9, 11, 13.

Quest. V. A great ship pursues a little one, steering the same way, at the distance of four leagues from it; and sails twice as fast as the small ship. It is asked how far the great ship must sail before it overtakes the lesser?

The distance of the two ships being 4, and their celerities being in a double Ratio, continue *in infinitum*, the double geometrical progression, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, &c, the first and the greatest terms of which is 4; and find the sum of all the infinite terms, by dividing 16 the square of the first 4, by 2 the difference of the two first, and the quotient 8 directs that the great ship must make 8 leagues before she can come up with the other.

S P O R T XIII.

Of Magical Squares.

BY a Magical Square, we understand a square divided into several other small equal squares, filled with terms of an arithmetical progression, so transposed, that all of the same line or rank, whether longitudinal, transverse, or diagonal, make the same sum.

This is the square here annexed, divided into 25 little boxes or squares, in

which the first 25 natural numbers are so transposed, that the sum of each rank from above downward, or from the right to the left, or along the diagonals or

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

diameters of the squares, is every way 65; which sum 65 is in all an odd square number, that is, it contains an odd square number of places, viz. 25, and is equal to the product arising from 5, the root of the square number 25; multiplied by 13, the middle term of the arithmetical progression, 1, 2, 3, 4, &c.

This sum is likewise found, by disposing the given terms of the arithmetical progression, according to their natural series, 1, 2, 3, 4, &c. in the square places, as you see here; for then the sum of each diagonal rank that is the rank

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

extending from one corner of the square to the other, is the sum demanded. This will likewise hold in even squares, or those which contain an even square number of boxes.

In order to dispose magically in the boxes of an odd square: for instance, that of 25 boxes, having 5 for its side; to dispose, I say, as many given numbers in arithmetical progression, as 1, 2, 3, 4, 5, and so on till you come to the last and greatest 25; write the first and the least immediately under the middle box, or that which possesses the center of the square; and moving diagonal wise to the right, write the second term 2 in the adjacent box, the lowermost of the next right hand rank. Here proceeding in the course of the diagonal from left to right you find no place for number 3, and so are to place it in the opposite or uppermost box of the rank into which it should have fallen. In like manner, finding no place for 4, you are to place it in the opposite

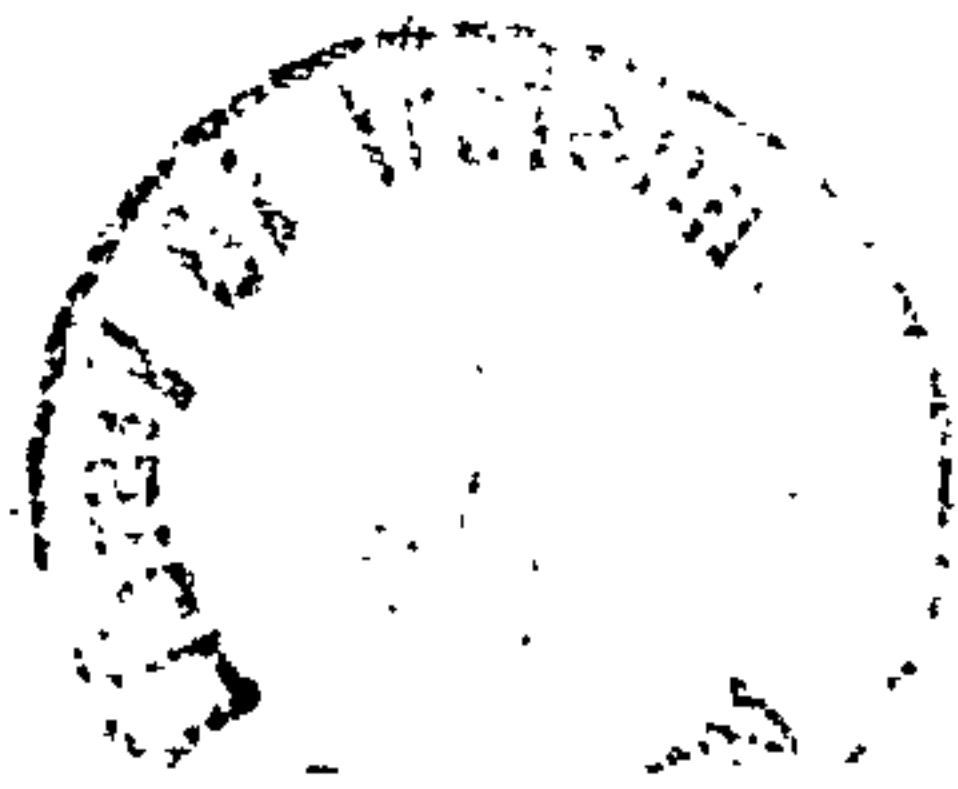
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site box of the rank that it falls to on the outside.

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Thus you continue proceeding still diagonal-wise to the right; but in regard 6 falls to a place that is already filled with 1, you must there take a retrograde diagonal course from the right to the left, and write 6 in the lowermost station of the rank in which the foregoing term 5 was placed, and so there will remain an empty place between 5 and 6. This retrograde course must always be observed when you fall in with a station already possessed. Continue to place the rest in order, according to these rules till you come to the angle of the square, where in this example stands 15: then, forasmuch as you can no longer move diagonal-wise to the right, you must place the term 16 in the second place (from the top) of the same rank; this done, the rest may be placed as the former, without any difficulty.

There are several magical dispositions both for odd and even squares; but these being difficult



difficult to understand, we reckon them improper for Arithmetical Recreations.

This square was called Magical, from its being in great veneration among the Egyptians, and the Pythagoreans their disciples, who to add more efficacy and virtue to this square, dedicated it to the seven planets divers ways, and engraved it upon a plate of the metal that sympathized with the planet. The square thus dedicated, was inclosed with a regular Polygon, inscribed in a circle divided into as many equal parts as there were units in the side of the square: with the names of the angles of the planet; and the signs of the zodiac written upon the void spaces between the Polygon and the circumference of the circle-circumscribed. Through vain superstition they believed that such a medal or Talisman would befriend the person that carried it about him upon occasion.

They attributed to Saturn the square of 9 places or boxes, 3 being the side, and 15 the sum of numbers in each row or column; to Jupiter the square of 16 places, 4 being the side, and 34 the sum of the numbers in each row; to Mars the square with 25, 5 being the side, and 65 the sum of numbers in each rank; to the Sun the square with 36, 6 being the side, and 111 the sum of each row; to Venus that of 49, 7 being the side, and 175 the sum of numbers in each rank or column; to Mercury that of 64, 8 being the side, and 260 the sum of each column; to the Moon the square with 81 lodges, having 9 for its side, and 369 for the sum of each column.

In fine, they attributed to imperfect matter, the square with 4 divisions, having 2 for the side; and to G O D the square of only 1 lodge, the side of which is an unit, which multiplied by itself, undergoes no change. By virtue of this Sport, we are taught to resolve the following Question.

Question. To draw up in three ranks the nine first cards, from an ace to a nine, in such a manner that all the points of each rank, taken either lengthwise or breadthwise, or diagonalwise, may make the same sum.

4	9	2
3	5	7
8	1	6

Dispose the nine first natural numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, magically, according to the directions laid down above; and as you see it done here, and place the cards according to their number, answerable to these figures.

8	256	2
4	16	64
128	1	32

1260	840	620
504	420	360
310	280	252

Instead of an arithmetical progression, you may take a geometrical; for instance, this double progression, 1, 2, 4, 8, 16, 32, 64, 128, 256, &c. and placing them magically, as above, you will find the product of each rank will be equal, viz. 4096, which is just the cube of the middle term 16.

Here we shall add by the by, one square more of 9 stations, in which the numbers

of each rank taken any away, as above, are in harmonical proportion; and you may find as many other numbers of the same quality, as you will, if instead of the foregoing numbers you put letters, as you see it done underneath, where the literal magnitudes of each rank are harmonically proportional; and so by giving different value to the three undetermined letters, a , b , c , you will have instead of literal quantities, numbers that will always preserve an harmonic proportion in each rank.

$$\begin{array}{rcl}
 a & & 2ac \\
 & & \hline
 2cb & & a+c \\
 \hline
 a+b & & c \\
 b & & \\
 & & \\
 & 2bc & 2abc \\
 & \hline & \hline
 & b+c & 2ab+ab-ba \\
 & 2abc & abc \\
 & & \hline
 2ac+ab-bc & & ab+ac-bc
 \end{array}$$

3	10	25	16	11	12	25	6	19	3
20	8	19	12	2	5	11	24	8	17
5	9	13	17	21	16	4	13	22	10
22	14	7	18	4	9	18	2	15	21
15	24	1	2	23	23	7	20	1	14

1	20	23	16	5	11	24	17	10	3
4	18	9	12	22	4	12	25	18	6
15	7	13	19	11	7	5	13	21	19
24	14	17	8	2	2	8	1	14	22
21	6	3	10	25	23	16	9	2	15

11	22	9	20	3	17	24	1	8	15
2	14	25	8	16	23	5	7	14	16
19	5	13	21	7	4	6	13	20	22
10	18	1	12	24	10	12	19	21	3
23	6	17	4	15	11	18	25	2	9

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the foregoing numbers; the Pyramidal numbers 1, 4, 10, 20, &c. formed by the continual addition of the triangular; the Pyramido-Pyramidals 1, 5, 15, 35, &c. formed by the continual addition of the Pyramidal; and so on, as you see in the following cut:

Among the different uses of the arithmetical triangle, I shall only single out those relating to Combinations, Permutations, and the Rules of Game; the rest being too speculative for Arithmetical Recreations.

By Combinations we understand all the different choices that can be made of several things, the multitude of which is known, by taking them divers ways, one by one, and two, three and three, &c. without ever taking the same twice.

For example, if you have four things expressed, by these four letters, *a, b, c, d*; all the different ways of joining two of them, as *ab, ac, ad, bc, bd, cd*; or three of them, as *abc, abd, acd, bad*; these I say, are called Combinations. And from hence it is easy to apprehend, that when four things are proposed, you may take them one by one four ways; two and two six ways; three and three four ways; and by fours only one way; so that 1 in \clubsuit combines four times; 2 six times; 3 four times; and 4 only once.

To find in a greater number of different things, such as seven; the divers combinations that may be made by taking them divers ways, whether by Addition or Multiplication: as, if
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you would know all the possible conjunctions of the seven planets, taking them two by two; that is to say, if you would know how often 2 combines in 7; add an unite to each of the two numbers given, 2, 7, and so you have 3, 8, which gives us to know, that in the third station (reckoning from below upwards, or from above downwards) of the eighth diagonal of the arithmetical triangle, you'll have the number of combinations demanded, *viz.* 21.

Or else, the two numbers given being 2 and 7, add together all the numbers of the second rank, till you come at the seventh diagonal, *viz.* 1, 2, 3, 4, 5, 6, and the sum 21 is what you want.

When the number of things proposed goes beyond 9, the triangle here delineated can't serve you; and therefore we shall give this general rule for any number whatsoever.

The two numbers given being 2 and 7, to know how often 2 the least will combine in 7 the greatest; make of them these two arithmetical progressions 2, 1, and 7, 6, which decrease by an unit, and ought to have but two terms, that is, as many as the least number 2 has units. Then multiply together all the terms of each progression, that is, 7 by 6, and 2 by 1; and divide the first product 42 by the second 2, and the quotient 21 satisfies the demand.

By this, or the foregoing method, you'll discover that three combines in 7, 35 times; 4 likewise 35 times; 5, 22 times; and 6 only 7 times. Whence it follows, that the number of
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all the combinations possible of seven different things, taken one by one, by two's, by threes, by fours, by fives, by sixes and sevens, amounts to 127, as appears by the addition of all the particular combinations, 7, 21, 35, 35, 21, 7, 1, which answer the numbers, 1, 2, 3, 4, 5, 6, 7. But you may find this total yet easier, by forming this double geometrical progression, 1, 2, 4, 8, 16, 32, 64, consisting of seven terms, answerable to the number of things combined, *viz.* 7; for the sum of these terms, 127, is the number you look for; which may still be found yet an easier way, *viz.* subtract 1 from the proposed number of things 7, and the remainder 6, directs you to take the sixth power (64) of the number 2; and the double of that power, bating an unit, 127, is the number desired.

Before I dismiss this subject, I shall here set down two methods peculiar to 2 and 3, for finding out how often these two numbers may be combined in any number of things. Suppose the number of things given is 7, you'll find how often 2 will combine in it, by subtracting the given number 7 from its square 49; and taking (21) the half of the remainder, 42, for the number desired. You'll find how often 3 may combine in 7, by adding 14, the double of 7 to 343, the cube of the same given number 7, and subtracting from the sum (357) the triple (147) of the square (49) of the same number (7) for then the sixth part (35) of the remainder (210) shews you, that three will combine in 7 35 times.

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There is another sort of combinations, that may be called Permutation, in which we take the same thing twice; as if you would combine these three numbers by two's, 2, 5, 6, in order to know what different quantities they can produce, if you consider the two first thus, 25, you'll call them twenty-five; if thus, 52, you'll call them fifty-two; in like manner, the first and third taken thus, 26, is a quite different quantity from the same two taken thus, 62; and so of all others. From whence it appears, that the multitude or number of permutations is the double of that of combinations.

Permutations are of very good use in making anagrams, and sometimes give very lucky hints; as in the word *ROMA*, the letters of which being transposed make this other word *AMOR*; but it is a much luckier hit that we meet with in these two Latin verses;

*Signa te, signa, temere me tangis & angis,
Roma tibi subito motibus ibit amor.*

the letters of which being read backwards, form the same verses.

We likewise make use of Permutations in playing at dice, to know the number of chances that attend the engaging to throw with two dice, 9 for instance; it being certain that the person who engages has four chances for it; for 9 may come up four ways, by *quatre cinque*, by *cinqe quatre*, by *tres six*, and again by *six tres*

tres (according as the first or second dye happens to appear).

To give the joint combinations of several letters, for example these four *AMOR*, that is to find the number of their simple permutations, by transposing them all possible ways; make this arithmetical progression, consisting of as many terms as there are letters to combine together, which in this example are four; so that the first term is always an unit, and the last denotes the number of letters; then multiply together all the terms, and the product 24 is the number of permutations or different changes that these four letters *AMOR* can undergo, as you see here;

A M O R	M A R O	O A M R	R O M A
A M R O	M A O R	O A R M	R O A M
A O M R	M O A R	O M A R	R M A O
A O R M	M O R A	O M R A	R M O A
A R M O	M R A O	O R A M	R A M O
A R O M	M R O A	O R M A	R A O M

By the same way do we find the number of Permutations of any other number of letters, *viz.* By making a progression of as many natural numbers as there are letters to combine, and multiplying together all the terms of the progression. Thus you will find that five letters may be transposed 120 ways; six 720; and so on as in the following table, where you see the 23 letters of the alphabet may be combined 25852016738984976640000.

1 | 1. A.

1	1. A.
2	2. B.
3	6. C.
4	24. D.
5	120. E.
6	720. F.
7	5040. G.
8	40320. H.
9	362880. I.
10	3628800. K.
11	39916800. L.
12	479001600. M.
13	6227020800. N.
14	87178291200. O.
15	1307674368000. P.
16	20922789888000. Q.
17	355687428096000. R.
18	6402373705728000. S.
19	121645100408832000. T.
20	2432902008176640000. V.
21	51090942171709440000. X.
22	1124000727777607680000. Y.
23	25852016738884976640000. Z.
24	620448401733239439360000.
25	1551121004333098598400000.

This table is easily calculated; for having discovered that four letters, for example, may be combined or transposed 24 ways; for if you multiply 24, the number of combinations, by 5 the next number, you have 120 for the combinations of five letters; and that multiplied by the next number 6, makes 720 for the combinations

nations of six letters; and so on through all the succeeding letters.

By Parti, in the way of gaming, we understand the just distribution or adjustment of what money out of the stakes belonging to several players, who play for it so many games, or a certain number of parts or sets, in proportion to what every one has ground to hope from fortune, upon the sets he wants to be up.

For example, If two gamesters have staked down forty pistoles, which is then no longer their property only by way of retaliation, they have a right to what chance may bring them, upon the conditions stipulated at the first agreement; suppose they were to play for these 80 pistoles three sets, that the first had gained one set, and the second none; that is, the first wants two sets to be out and the second three; these suppositions being laid down, and the gamesters having a mind to draw their stakes, without standing to their chances, the just Quota appertaining to each, is what is called Parti, and is found out by the arithmetical triangle, after this manner:

Since the supposition runs, that the first gamester wants 2 sets, and the other three, and the sum of the two numbers 2 and 3 is 5; we must turn to the fifth diagonal of the arithmetical triangle, and there take five the sum of the two first numbers 1, 4, by reason of the two sets that the first gamester is short; and 11 the sum of the other three, 6, 4, 1, by reason of the three sets that the second gamester is short: and

E

these

these two sums 5 and 11 give the reciprocal Ratio of the two Parti's inquired for; so that the Parti or Quota of the one or first is to that of the second, as 11 to 5.

But to adjust these Quota's, that is, to assign each gamester his positive share of the 80 pistoles at stake, this number 80 must be divided into two parts proportional to the two terms 11, 5; and this is done by multiplying 80 by the two sums 11, 5, separately, and dividing each of the two products, (880, 400) by 16, the sum of the two terms 11, 5; by which means you have 55 for the number of pistoles due to the first gamester that gained a set; and 25 for the other that gained none.

In like manner, if the first wants but 1 set to be out, and the second 2, we add together these two numbers 1, 2, and their sum being 3, turn to the third diagonal of the arithmetical triangle, and there take the first number 1, and the sum 3 of the two others 2, 1; from these two numbers 1, 3, we learn that the first his Quota is to that of the second as 3 to 1; and since the sum of these two terms is 4, the consequence is, that the first gamester ought to have $\frac{3}{4}$ of the 80 pistoles staked, and the second only $\frac{1}{4}$, that is, the first 60 pistoles, and the other 20.

Hence it appears, that when the game is at this pass, the first may lay upon the square 3 to 1: and this we can likewise make out without the arithmetical triangle, after the following manner:

Since

Since the first wants one set to be out, and the second two, we must consider, that if they went on with the game, and the second gained a set, then the two gamesters would have equal chances, and so their quota's or dividends would be equal, it being a constant and a general rule that the one share of the first is to that of the second, as the chances of the one are to those of the other. And so in this supposition, each of them has a title to an equal half of the money. 'Tis therefore certain, that if the first gains the set that is to be played, he sweeps all; but if he loses it, he has a title to an equal half: and therefore, if they have a mind to draw without playing the set, the first ought to have half the money at stake, and the half of the remaining half, that is $\frac{3}{4}$ of the whole; so that $\frac{1}{4}$ remains to the second, for it is evident, that if a gamester has a right to a certain sum, in case he gains, and to a lesser in case he loses, he has a right to the half of those two taken together, if the game is thrown up.

This first case directs us to the solution of the second, which supposes the first to want one set to be out, and the second three; for if the first gains the set, he sweeps all the 80 pistoles; if he loses, it turns to the first case as above, that is, he has a right only to $\frac{3}{4}$; and therefore, if the stakes are drawn without playing that set, his right is half of these two sums taken together, *i. e.* $\frac{7}{8}$ or 70 pistoles, $\frac{1}{8}$ or 10 pistoles remaining to the second.

This leads us to a resolution of a third case. Supposing the first to be two sets short, and the second three; for if the first gains the next set, he has a right to $\frac{7}{8}$ of the money, by the second case; if he loses it, so that the second case wants only two to be out, as well as he, the money is to be equally divided between them. Upon the whole, the game stands thus; if the first wins, he claims $\frac{7}{8}$, if he loses, he claims $\frac{1}{2}$; and therefore, if the game is thrown up without playing this set, he claims the half of these two sums put together, *i. e.* $\frac{5}{8}$ or 55 pistoles, leaving $\frac{3}{8}$ or 25 to the second.

The second case leads us likewise to the solution of a fourth case, in which the first is supposed to be one set short of the whole, and the second four; for if the first gains a set, he carries the 80 pistoles; if he loses it, so that the second lacks only three to be out, he claims $\frac{7}{8}$ by the second case. Now since, in case of winning, he takes 80 pistoles, and in case of losing $\frac{7}{8}$ of them, his dividend, upon throwing up, is the half of these two sums put together, that is, $\frac{5}{8}$, or 75 pistoles, and so he leaves $\frac{3}{8}$, or 5 pistoles for the second.

The fourth and third cases leads us after the same manner, to the solution of a fifth, which supposes that the first gamester is two sets short, and the second four; for if the first gains a set, and so lacks but one to be out, he claims $\frac{5}{8}$ by the fourth case; and if he loses it, so that the second wants but three, he claims $\frac{1}{2}$ by the third; and consequently, in case of drawing, his

his due is the half of these two sums put together, that is, $\frac{1}{2} \times \frac{3}{8}$, or 65 pistoles, $\frac{1}{2} \times \frac{3}{8}$, or 15 pistoles being left for the second. And so of the other cases.

All these, and an infinite number of other cases that may happen, are solvable without the arithmetical triangle, after a different and an easy manner, as follows:

Take the fifth case for instance, which supposes the first to be two sets short, and the second four; in this supposition the two gamesters want between them six sets to be out: Take 1 off the six, and since the remainder is 5, suppose these five letters of the same form *a a a a a*, to favour the first gamester; and these five *b b b b b*, to favour the second; make combinations of these ten letters, as you see it here done; where of 32 combinations, the first 26 to the left, having at least two *a*, are taken for the number of chances that can make the first to win; because he lacks two sets, and the remaining 6 to the right, or where there are at least four *b*, are taken for the number of chances upon which the second may win; because he wants four to be out.

<i>aaaaa</i>	<i>aaabb</i>	<i>aabbb</i>	<i>abbbb</i>
<i>aaaab</i>	<i>aubba</i>	<i>abbba</i>	<i>abtba</i>
<i>aaaba</i>	<i>abbaa</i>	<i>bbbaa</i>	<i>babbb</i>
<i>aabaa</i>	<i>btaaa</i>	<i>ababb</i>	<i>bbatb</i>
<i>abaaa</i>	<i>aabab</i>	<i>abbab</i>	<i>bbbab</i>
<i>baaaa</i>	<i>abaab</i>	<i>ababb</i>	<i>bbbbb</i>
	<i>baaab</i>	<i>baabb</i>	
	<i>baaba</i>	<i>babba</i>	
	<i>babaa</i>	<i>bbaba</i>	
	<i>ababa</i>	<i>babab</i>	

Thus it is plain, that the first his due is to that of the second as 26 to 6, or, as 13 to 3.

In like manner to solve the third case, which supposes the first to want two sets to be up, and the second three, so that they want five between them; take 1 from the said sum 5, and since the remainder is 4, suppose these similar letters *a a a a* to be favourable to the first, and these four *b b b b* to the second, and combine these eight letters together, as you see it here done;

where, of the 16 combinations, the first 11 to the left, having at least two *a*'s, must represent the number of chances that the first has for game, two sets being what he wants; and the re-

<i>aaaa</i>	<i>aabb</i>	<i>abbb</i>
<i>aaab</i>	<i>abba</i>	<i>bbba</i>
<i>aaba</i>	<i>bbaa</i>	<i>bbab</i>
<i>abaa</i>	<i>baab</i>	<i>babb</i>
<i>baaa</i>	<i>baba</i>	<i>bbbb</i>
	<i>abab</i>	

maining 5 to the right, having at least three *b*'s, must be taken for the number of chances that can make the second up, he being three sets short. Thus the claim of the first is to that of the second, as 11 to 5, &c.

The same 16 combinations will serve for the solution of the fourth case, in which the first was supposed to be one set short, and the second four; so that five is the number of sets wanted between them, as in the third case. For among these 16 you will find 15 that have at least one *a*, (answerable to the one set that the first wants) for the chances upon which the first will win; and only one that has four *b*'s, the second being four sets short, which shews there is but one chance that can save the second. Thus the first

first share is to that of the second, as 15 to 1. And so of all other cases.

To know, when two are at play, what advantage one has, that engages to throw 6, for example, with one dye at a certain number of throws, and first of all, at the first throw; we must consider that his case is 1 to 5; for he has but one chance to win, and five to lose upon; and consequently if he lays upon one throw, he ought to lay but 1 to 5.

To engage to throw 6 with one dye at two throws, is the same thing, as to throw two dyes at a time, one of which is to be a 6; and in that case, he who throws has but 11 chances to win upon, since he may throw the first 6, and the second 1, 2, 3, 4, or 5; or the second 6, and the first 1, 2, 3, 4, or 5; or else both dyes fixes; whereas he has 25 to lose upon, as you see here. Where

it is easy to conclude, that he who offers to throw with one dye at two throws, ought to set but 11 to 25.

1	1	2	1	3	1	4	1	5	1
1	2	2	2	3	2	4	2	5	2
1	3	2	3	3	3	4	3	5	3
1	4	2	4	3	4	4	4	5	4
1	5	2	5	3	5	4	5	5	5

When you lay upon 6 at two throws, take notice that 36, the sum of all the chances, 11, 25, is the square of the given number 6; and that 25, the number of chances against him who throws, is the square of the same number, wanting 1, that is, 5. And therefore to find the number of chances that favour him who is to throw, you need only take 1 from 12, the

double of the number given, and the remainder 11 is the number required; which being subtracted from 36, the square of the former number 6, leaves 25 the remainder, which will always be a square number, and denote the chances against him.

To lay upon 6 at three throws with one dye is the same as to lay upon 6 at one throw with three dice; and in that case, he who throws has 91 favourable chances, and 125 against him, and so ought to set but 91 to 125; thus you see he is at a loss who lays upon the square for 6 at three throws of one dye.

Take notice that the sum 216 of all the chances 91, 125, is the cube of the given number 6, when you engage to throw 6 at three throws with one dye; and that 125, the number of the chances against you is the cube of the same number given, less 1, *i. e.* 5. And therefore, to find the number of chances that favour the person that throws, you need only to subtract 125, the cube of the given number 6, wanting 1 (*i. e.* 5) from 216 the cube of the same number given.

By the same method we find out what advantage he has who proffers to throw 6 with one dye at four throws; for if we subtract from the fourth power or biquadrate 1296 of the given number 6, if we subtract, I say, from that, 625 the biquadrate of the same number, less one, or of 5, the remainder shews us 671 favourable chances for him that throws; the biquadrate 625 being the number of the chances against him:

him: So that he who lays upon 6 at four throws has the odds on his side.

But he has a much greater advantage upon 6 at five throws with one dyce, as appears by subtracting 3125, the fifth power of 5 (the given number, bating 1) from 7776, the fifth power of the given number 6; for the remainder 4651, is the number of the favourable chances, and 3125, the fifth power subtracted, is the number of those against him who throws.

If you want to know what advantage he has who offers with two or several dice, to throw at one throw a determined raffle; for example two *trav*; you must consider, that with two dice he has but one chance to save him, and 35 to loose upon, since two dice can combine 36 different ways, that is, their 6 faces may have 36 different postures, as you see by this scheme;

1	1	2	1	3	1	4	1	5	1	6	1
1	2	2	2	3	2	4	2	5	2	6	2
1	3	2	3	3	3	4	3	5	3	6	3
1	4	2	4	3	4	4	4	5	4	6	4
1	5	2	5	3	5	4	5	5	5	6	5
1	6	2	6	3	6	4	6	5	6	6	6

This number 36, is the square of 6, the number of faces, there being but two dice; but if there were three, the cube of 6, 216, would be the number of combinations: and if there were four, the biquadrate of 6, 1296 would be the number. And so on.

From

From what has been said, it is evident, that in engaging a determined raffle at one throw with two dice, one ought to lay but 1 to 35; and by a parity of reason, that he ought to lay 3 to 213 upon a determined raffle or pair-royal with three dice; and 6 to 1290 with four; for of the 216 chances of three dice, there's only three that can favour him, since three things can combine by two's only 3 ways; and of 1296 chances of four dice, only 6 can favour the thrower, since four things combine by two's 6 ways.

But if you want to know what odds he lies under who proffers to throw a raffle of one sort or t'other at the first throw of two or more dice; you may find without difficulty, that he ought to set but 6 to 30, or 1 to 5 upon two dice, since of the 36 chances of two dice, there is only 6 that can make a raffle; and that upon three dice, his case is 18 to 198, or 1 to 11, since of the 216 chances, that three dice can fall upon only 18 can produce a raffle.

S P O R T X V .

Several dice being thrown, to find the number of points that arise from them, after some operations.

SUPPOSE three dice thrown upon a table, which we shall call A, B, C, bid the person that threw them add together all the uppermost points, and likewise those underneath of any two

two of the three: for instance, B and C; A being set apart, without altering its face. Then bid him throw again the same two dice, B and C, and make him add to the foregoing sum all the points of the upper faces, and withal the lowermost points, or those underneath of one of them, C for instance, B being set apart near A, without changing its face, for giving a second sum. In fine, order him once more to throw the last dye C, and bid him add to the foregoing second sum the upper points, for a third sum, which is thus to be discovered. After the third dye C is set by the other two, without changing its posture, do you come up, and compute all the points upon the faces of the three dice, and add to their sum as many 7's as there are dice, that is, in this example 21, and the sum of these is what you look for; for when a dye is well made, 7 is the number of the points of the opposite faces.

To exemplify the matter: Suppose the first thrown of the three dice, A, B, C, brought up 1, 4, 5; setting the first 1 apart, we add to these three points, 1, 4, 5, the points 3 and 2 that are found under or opposite to the upper points 4 and 5 of the other two dice; and this gives me the first sum 15. Now suppose again that the two last dice are thrown, and shew uppermost the two points 3 and 6, we set that with the three point apart, near the die that had 1 before, and add to the foregoing sum (15) these two points 3 and 6, and withal 1 the point that is found lowermost in the die that is still kept in service,

service, and had 6 for its face at this throw; thus we have 25 for the second sum. We suppose at last, that this third and last die being thrown a third time, it comes up 6, which we add to the second sum 25, and so make the third sum 31. And this sum is to be found out by adding 21 to 10 the sum of the points 1, 3, 6, that appear upon the faces or uppermost sides of the three dice then set by.

S P O R T X V I.

Two dice being thrown, to find the upper points of each die without seeing them.

MAKE any one throw two dice upon a table, and add 5 to the double of the upper points of one of them, and add to the sum multiplied by 5, the number of the uppermost points of the other or the second die; after that, having asked him the joint sum, throw out of it 25, the square of the number 5 that you gave to him, and the remainder will be a number consisting of two figures; the first of which to the left representing the tens, is the number of the upper points of the first die, and the second figure to the right representing units, is the number of the upper points of the second die.

We will suppose that the number of the points of the first die that comes up is 2, and that of the second 3; we add 5 to 4, the double of the points of the first, and multiply the sum 9 by the same number 5, the product of which

which operation is 45, to which we add 3, the number of the upper points of the second die, and so make it 48, then we throw out of it 25, the square of the same number 5, and the remainder is 23, the first figure of which 2 represents the number of points of the first die, and the second 3 the number of points of the second die.

Another way of answering this Sport is this; ask him who threw the dice, what the points underneath make together, and how much the under points of one surpass those of the other; and if this excess is, for example, 1, and the sum of all the lower points is 9, and these two numbers 1 and 9, and subtract the sum 10 from 14; then take 2, the half of the remainder 4, for the number of the upper points of one of the dice; and as for the other die, instead of adding the excess 1; to the sum 9, subtract it out of 9, and take the remainder 8 out of 14, 6 is the remainder, the half of which 3, is the number of the upper points of the second die.

A third way is this: bid the person who threw the dice, add together the upper points, and tell you their sum, which we here suppose to be 5; then give him orders to multiply the number of the upper points of one die by the number of upper points of the other die; and to acquaint you in like manner with their product, which we here suppose to be 6: now having this product 6, and the preceding sum 5, square 5, and from its square 25 subtract 24, the quad-

quadruple of the product 6, and the remainder is 1: then take the square root of the remainder, which in its case is 1, and by adding it to, and subtracting it from the foregoing sum 5, you have these two numbers 6, 4, the halves of which, 3, 2, are the numbers of the upper points of each die.

SPORT XVII.

Upon the throw of three dice, to find the upper points of each die, without seeing them.

ORDER the person that has thrown the dice, to place them near one another in a straight line, and ask him the sum of the lowermost points of the first and second die, which we here suppose to be 9; then ask him the sum of the points underneath of the second and third, which we here suppose to be 5; and at last the under points of the first and third, which we put 6. Now, having these numbers given you, 9, 5, 6, subtract the second number 5 from 15, the sum of the first and third, 9 and 6; and the remainder 10 from 14; so there remains 4, the half of which 2 is the number of the upper points of the first die. To find the number of the upper points of the second, subtract the third number 6 from 14, the sum of the two first 9 and 5; and the remainder 8 from 14 again; so you have a second remainder 6, the half of which, 3, is the number demanded. At last for the third die, subtract the first

first number 9 from 11, the sum of the second and third 5, 6, and the remainder 2 from 14; so you have a second remainder 12, the half of which, 6, is the number of the upper points of the third die.

S P O R T XVIII.

To find a number thought of by another.

ORDER the person to take 1 from the number thought upon, and after doubling the remainder, to take 1 from it, and to add to the last remainder, the number thought upon. Then ask him what that sum is, and after adding 3 to it, take the third part of it for the number thought of. For example, let 5 be the number, take 1 from it, there remains 4; then take 1 from 8, the double of that 4, and the remainder is 7, which becomes 12, by the addition of 5, the number thought of; and that 12, by the addition of 3, makes 15, the third part of which, 5, is the number thought of.

Another way is this. After taking 1 from the number thought of, let the remainder be tripled; then let him take 1 from that triple, and add to the remainder the number thought of. At last, ask him the number arising from that addition, and if you add 4 to it, you will find the fourth part of the sum to be the number thought of. Thus 5, bating 1, makes 4, that tripled makes 12, which losing 1 sinks to

11, and enlarged by the accession of 5, comes to 16, which, by the addition of 4, is 20, and the fourth part of that, *viz.* 5, is the number thought of.

3d way. Add 1 to the number thought of, double the sum, and add 1 more to it, and then add to the whole sum the number thought of. Having learned the sum total, take 3 from it, and the third part of the remainder is what you look for. Thus, 5 and 1 is 6, and the double of that, enlarged by 1 is 13, which by the addition of 5, comes to 18; take 3 from that, the remainder is 15, the third part of which, 5, is the number thought of.

4th. Or else after adding 1 to the number thought of, bid the person triple the same, and add first 1 to it, and then the number thought of. At last, ask the sum of this last addition, and after robbing it of 4, take the fourth part of the remainder for the number thought of. Thus, 5 and 1 is 6, the triple of which and 1 is 19, which, with 5 is 24, and that bating 4 is 20, the fourth part of which, 5, answers the Sport.

5th. Take 1 from 5, the number thought of, double the remainder 4, from which, 8, take 1, and likewise the number thought of; after which, ask for the remainder 2, and add 3 to it, so you have your number.

6th. Let the person that thinks add 1 to the five, the number thought of, and to the double of that, 12, 1 more, and subtract from the sum 13, the number thought of; then ask for the remainder

mainder 8, and taking 3 from it, what you leave behind, 5, is the number thought of.

7**th**. Bid the person that thinks take 1 from 5, the number thought of; and 1 from 12, the triple of the remainder; and then the double of the number thought of; 10 from 11, the last remainder. This done, ask for the remainder of the third subtraction, *viz.* 1, and adding 4 to it, you will find satisfaction.

8**th**. Add 1 to the number thought of 5, adding 1 more to the triple of that you have, 19, from which take 10, the double of the number thought of; then ask for the remainder, 9, from which take 4, and so you are right.

9**th**. Order the person to tripple the number thought of, (5) and out of the triple number (15) to cast away the half, if it were possible; and since in this example it is not, to add 1 to it so as to make it 16; the half of which 8, must be trippled, and that makes 24. The person that thinks having done this, ask him how many 9's are in the last triple (24); he answers two; so you are to take 2 for every 9, which in this example makes 4, and by reason of the 1 you gave to make the 15 an even number, you are here to repay it by addition to the 4, and so you have 5, the number thought of. If there happens to be no 9 in the last triple, the number thought of is 1.

10**th**. Bid him add 1 to the number thought of (which makes 6); then subtract it from it, and so it leaves (4) a remainder; then bid him multiply the sum (6) into the remainder (4)

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and

and tell you the product. To this product 24 add 1, and of the sum 25, take the square root 5:

11th. Bid the person that thinks add 1 to the number thought of (which we all along suppose to be 5) and multiply the sum (6) by the number thought of (5), then let him subtract the number thought of (5) from the product (30) and tell you the remainder (25) the square root of which 5 is the number thought of.

12th. After taking 1 from the number thought of, bid him multiply the remainder (4) by the number thought of (5) and add to the product (20) the same number thought of, and tell you the sum 25, of which you are to extract the square root 5.

13th. Bid him add 2 to the number thought of, and clap a cypher to the right of the sum, which makes 70; and to that add 12, to the sum of which addition (82) let him clap another cypher, so as to make it 820. From this decuple (820) let him subtract 320, and tell you the remainder 500, from which you are to cut off the two cyphers (each of which did still decuple the number it was put to) and so you have the number thought of 5.

14th. Let him add 5 to the double of the number thought of; to the sum 15 let him add a cypher on the right-hand to decuple it; then let him add 20 to the sum (150) and to the last sum (170) let another decupling cypher; at last let him subtract 700 from the last sum of all
(1700)

(1700) and discover to you the remainder 1000, which from you are to strike off two cyphers to the right, and take the half of the remainder (10) for the number thought of.

These two last methods are not very subtle; for the last number being known, it is an easy matter, by a retrograde view, to find out the other numbers, and by consequence the number thought of. And upon that consideration we shall here subjoin two other methods that are more myfterious.

15th. Bid the person that thinks add 1 to the triple of the number thought of, and triple the sum (16) again: to which last sum (48) bid him add the number thought of (5); then ask him the sum of all (53) and from that take off 3, and the right-hand cypher from the remainder 50; which leaves you 5 to the left for the number thought of.

16th. Bid him take 1 from the triple of the number thought of (15) and multiply the remainder (14) by 3; and add to (42) the product, the number thought of (5); then ask the sum of the addition, 47, to which add 3, and cut off from the sum 50 the cypher, which must needs be on the right-hand, and so leaves to the left the number thought of.

From these two last methods we may draw this inference, that *if we add an unit to the triple of any number (as to 18 the triple of 6) and the same number (6) to the triple of the sum (57) the second sum (63) will always terminate with 3.*

Another inference is, that *if we subtract an unit from [18] the triple of any number [6] and add the same number [6] to the triple of the remainder (51 the triple of 17) the sum [57] will always end with the figure 7.*

The last inference is, that this double Sport is impossible, viz. *To find a number of such a quality, that if you add to, or subtract from its triple, an unit, and add the same number to the triple of the sum of the remainder, the last sum will be a perfect square number; for as we shewed in Sport 9 no number ending in 3 or 7 can be a true square. See the following Sport.*

S P O R T X I X .

To find the number remaining after some operations, without asking any questions.

LET another think of a number at pleasure; bid him add to the double of it an even number, such as you have a mind to. For example 8; then bid him subtract from half the sum the number thought on, and what remains is the half of the even number that you ordered him to add before; and so you may roundly tell him you are sure the remainder is 4.—
 Though the demonstration of this is easy, yet those who are not apprised of the reason will be surpris'd at it. However, that you may light exactly on the number thought of, conceal your knowledge of the remainder 4, and bid him subtract that remainder, whatever it is, from

from the number thought of, if so be it be larger ; or else if the number be less, to subtract it from the remainder ; and then ask him for the remainder of the last subtraction ; for if you add this remainder to the half of the even number you gave him (*i. e.* 4 the half of 8) when the number thought of is larger than that of the half of the even number ; or if you subtract the remainder from the same half [4] when the number thought of is less than it, you will have the number thought of. To exemplify the matter, let 5 be the number thought of, and 8 added to its double 10, which makes 18 ; the half of that is 9 ; and 5, the number thought of, subtracted from 9 leaves 4, the half of the additional number 8 ; and if you take this half 4 from the number thought of 5, there will remain 1, which being added to the same half 4 (the number thought of being greater than that half) gives 5, the number thought of. In like manner if to 10, the double of 5 the number thought of, you add 12, you will have 22, the half of which is 11 ; and from thence taking the number thought of 5, there remains 6, the half of the additional number 12 : and if from that half 6, you take the number thought of 5, (which in this example is less than the said half) there will remain 1, which being taken from the same half, since the number thought of is less than that half [6] leaves 5 for the number thought of.

But an easier way to answer the sport is this : Bid the person that thinks, take from the dou-

ble of the number thought of, any even number you will that is less, for example 4; then let him take the half of the remainder from the number thought of, and what remains will be 2, the half of the first number subtracted 4; and therefore to find the number thought of, bid him add the number thought of to that half 2, and then ask the sum, 7, from which you are to take the same half, and so there will remain 5 for the number thought of.

But another, and yet easier way is this: Bid him add what number you will to the number thought of, and multiply the sum by the number thought of; for if you make him subtract the square of the number thought of from the product, and tell you the remainder, you have nothing to do but to divide that remainder by the number you gave him to add before; for the quotient is the number thought of. Thus 4 added to 5 (the number thought of) makes 9, which being multiplied by 5, makes 49; from which take 25, the square of the number thought of, and there remains 20, which being divided by 4, leaves 5 in the quotient.

Or else bid the person that thinks, take a certain lesser number from the number thought of, and multiply the remainder by the same number thought of; for if you make him take the square of the number thought of from the product, and tell you the remainder; by dividing that remainder by the number you ordered to be taken from the number thought of, you

you have the number thought of in the quotient.

But of all the ways for finding out a number thought of, the following is certainly the easiest; make him take from the number thought of what number you pitch upon that is less than it, and set the remainder apart; then make him add the same number to the number thought upon, and the preceding remainder to the sum, for a second sum; which he is to discover to you, and the half of that sum is the number thought of. Thus 5 being thought of, and 3 taken from it, the remainder is 2; and the same number 3 added to 5, makes 8, and that, with the preceding remainder, 10, the half of which, 5, is the number thought of.

S P O R T XX.

To find the number thought of by another, without asking any questions.

B ID the other person add to the number thought of, its half if it be even, or its greatest half if it be odd; and to that sum its half or greatest half, according as it is even or odd, for a second sum, from which bid him subtract the double of the number thought of, and take the half of the remainder, or its least half, if the remainder be odd; and thus he is to continue to take half after half, till he comes to an unit. In the mean time you are to ob-

serve how many subdivisions he makes, retaining in your mind for the first division 2, for the second 4, for the third 8, and so on in a double proportion, remembering still to add 1 every time he took the least half; and that when he can make no subdivision, you are to retain only 8. By this means you have the number that he has halved so often, and the quadruple of that number is the number thought of, if so be he was not obliged to take the greatest half at the beginning, which can only happen when the number thought of is evenly even, or divisible by 4; in other cases, if the greatest half was taken at the first division, you must subtract 3 from that quadruple; if the greatest half was taken only at the second division, you subtract but 2; and if he took the greatest half at each of the two divisions, you are to subtract 5 from the quadruple, and the remainder is the number thought of.

For example, let 4 be the number thought of, which by the addition of its half 2, becomes 6, and that, by the addition of its half 3, is 9: from which 8, the double of the number thought of, being subtracted, the remainder is 1, that admits of no division; and for this reason you retain only 1 in your mind, the quadruple of which, 4, is the number thought of.

Again; let 7 be the number thought of; this being odd, the greatest half of it, 4, added to it makes 11, which is odd again; and so the greatest half of 11 added to 11, makes 17, from which we take 14, the double of the number
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ber thought of, and so the remainder is 3, the least half of which is 1, that admits of no further division. Here there being but one subdivision, we retain 2, and to that add 1 for the least half taken, so we have 3, the quadruple of which is 12. But because the greatest moiety was taken both in the first and second division, we must subtract 5 from 12, and the remainder 7 is the number thought of.

S P O R T XXI.

To find out two numbers thought of by any one.

HAVING bid the person that thinks add the two numbers thought of (for example 3 and 5) order him to multiply their sum [8] by their difference [2] and to add to the product [16] the square [9] of the least of the two numbers [3] and tell you the sum, 25, the square root of which 5, is the greatest of the two numbers thought of. Then for the least, bid him subtract the first product [16] from the square [25] of the greatest number thought of [5] and tell you the remainder, 9, of which the square root 3 is the least number thought of.

An easier way of doing it is this: Bid him add to the sum of the two put together [8] their difference [2] and tell you the last sum, 10, for the half of it, 5, is the greatest number thought of. And as for the least, bid him subtract the difference of the two numbers thought of from their sum, and ask him the remainder, 6, the

6, the half of which, 3, is the number you look for.

This Sport may likewise be solved after the following manner: Bid him square the sum of the two numbers (*which is 64 in this example*) then bid him add to the least number thought of [3] the double [10] of the greatest [5] and multiply the sum [13] by the least [3] and subtract the product [39] from the foregoing square [64] and discover the remainder 25, the square root of which is the greatest number thought of; and as for the least, order him to add to the greatest [5] the double [6] of the least [3], and multiply the sum [11] by the greatest [5] and subtract the product 55, from the foregoing square [64] and tell you the remainder [9] the square root of which is 3, the least number thought of.

Another, and a very easy way, is this: bid him multiply the two numbers [5, 3] together; and then multiply the sum of the two numbers [8] by the number you want to find, whether the greater or lesser, and subtract the product of the two numbers [15] from that product (which is 40, if you want the greater, and 24, if you look for the lesser number) and tell you the remainder, 25, or 9, the square roots of which satisfies the demand.

Or else, bid him first take the product of the two numbers [15], then multiply their difference [2] by the number enquired for [3 or 5] and add to that product the product of the two numbers [15] if you want the greatest, or subtract

tract that product from the product of the two numbers, if you look for the least. Then he telling you the sum, or the remainder, their square roots are the numbers in question.

When the least of the two numbers does not exceed 9, it is easy to find them out after this manner; let one be added to the tripple of the greatest, and the two numbers thought of to the tripple of that sum, and the total sum discovered; from which you are to take off 3, and then the right-hand figure is the least, and the left-hand figure the greatest number thought of. Thus 5 and 3 being thought of, 1 added to the tripple of 5, is 16, and the tripple of that [48] added to 8, the sum of the two numbers, makes 56, which losing 3, is 53; 3 the right-hand figure being the least, and 5 on the left the greatest number thought of.

S P O R T XXII.

To find several numbers thought on by another.

IF the quantity of numbers thought of is odd, ask for the sums of the first and second, of the second and third, of the third and fourth, and so on till you have the sum of the first and last; and having written all these sums in order, so that the last sum is that of the first and last; subtract all the sums of the even places from all those in the odd places; and the half of the remainder is the first number thought of, which being subtracted from the first sum, leaves the second

second number remaining, and that subtracted from the second, leaves the third number remaining, and so on to the last. For example; suppose these five numbers thought of, 2, 4, 5, 7, 8, the sums of the first and second, of the second and third; and so on to the sum of the first and last, and 6, 9, 12, 15, 10: and 24 the sum of the even places, 9 and 15, being taken from 28, the sum of the odd places, there remains 4, the half of which, 2, is the first number thought of, and that being taken from the first number 6, leaves 4 for the second number, and 4 taken from the second, 9, leaves 5 for the third, and so on.

If the quantity of numbers thought upon is even, ask for the sums of the first and second, of the second and third, of the third and fourth, and so on to the sum of the second and last; write them all in order, so that the sum of the second and last may be the last in order: take all the sums in the odd places (excepting the first) from those in the even, and the half of the remainder is the second number thought of, and that taken from the first sum, leaves the first number, which taken from the third sum, leaves for a remainder the third number, and so on. Thus 2, 4, 5, 7, 8, 9, being the numbers thought of, the sums proposed, as above, are 6, 9, 12, 15, 17, 13. Then take 29 the sum of 12 and 17 the odd places (excepting the first) out of 37 the sum of 9, 15, 13, the three even stations, and the remainder is 8, the half of which, 4, is the second number thought of; and

and that taken from 6, the first sum, leaves 2 the first number, as the same second number 4, taken from the second sum 9, leaves 5 for the third number, which taken from the third sum 12, leaves 7 for the fourth, and so on.

When each of the numbers thought of consists only of one figure, they are easily found in the following manner: let the person add 1 to the double of the first number thought of, and multiply the sum by 5, then add to the product the second number thought of. If there is a third number, add 1 to the double of the preceding sum, and after multiplying the whole by 5, add to the product the third number thought of. In like manner, if there is a fourth number, bid him add 1 to the double of the last preceding sum, and after multiplying the whole by 5, add to the product the fourth number thought of, and so on, if there are more numbers. This done, ask the sum arising from the addition of the last number thought of, and subtract from it 5 for two, 55 for three, and 555 for four numbers thought of, and so on, if there are more; and then the first left-hand figure of the remainder is the first number thought of, the next (moving to the right) is the second, the next to that the third, and so on till you come to the last right-hand figure, which is the last number thought of.

For example, let 3, 4, 6, 0, be the numbers thought of, and 1 added to 6, the double of the first 3, and the sum 7 multiplied by 5, the

the product of which, 35, with the addition of the second number 4, is 39; then 1 being added to 78, the double of 39, and the sum 79 multiplied by 5, the product 395, with the addition of the third number 6, is 401; and the double of that, with the addition of an unit is 803, which multiplied by 5 is 4015, and with the addition of the fourth number, 9, 4024. Now, if from this sum 4024, we take 555, the remainder is 3469, the four figures of which are the four numbers thought of.

But there is a method for this purpose that is still easier, *viz.* Let 1 be subtracted from the double of the first number, and the remainder multiplied by 5, to the product of which multiplication, let the second number thought of be added, then, if there be more numbers than two let him add 5 to the last sum for a second sum; let 1 be taken from the double of this second sum, and the remainder multiplied by 5, and the third number added to that product; this done, if there are no more numbers thought of (otherwise you must add 5, and go on again) ask for the last sum, add 5 to it, and the figures of the whole sum will represent the numbers thought of, as above.

For instance, let 3, 4, 6, 9, be thought of; take 1 from 6, the double of the first 3, multiply the remainder 5 by 5, add to the product 25, the second number 4; to the sum 29 add 5, which gives you 34 for a second sum, take 1 from 68, the double of this second sum, multiply the remainder 27 by 5, and to the product

product 335, add the third number 6, which makes 341; add 5 to this last sum, then it makes 346, the double of which, wanting 1, is 691, and that multiplied by 5, 3455, which with the addition of the fourth number 9, is 3464. Now adding 5 to this sum, you have 3469, the four figures of which represent the four numbers thought of.

S P O R T XXIII.

A person has in one hand a certain even number of pistoles, and in the other an odd number; it is required to find out in which hand is the even or the odd number?

LET the number in the right-hand be multiplied by any even number you will, as 2, and the number in the left by such an uneven number as you pitch upon, as 3, then order the person to add together the two products, and take the half of their sum, and if he can take an exact half, so that the sum is even, you will know by that, that the number in the right-hand being multiplied by an even number is odd, and consequently that in the left multiplied by an odd number is even. But on the contrary, if he cannot take an exact half, the number in the right is even, and that in the left odd.

For example, suppose 9 pistoles in the right-hand, and 8 in the left; multiply 9 by 2, and 8 by 3; the sum of the two products 42 being
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an even number, shews that 9, the odd number, multiplied by the even 2, is in the right-hand, and consequently 8 the even in the left. This Sport directs us to the solution of the following question:

Question. A man having a piece of gold in one hand and silver in the other, it is asked in what hand the gold or silver is in?

Fix a certain value in an even number, as 8, on the gold; and an odd, as 5, upon the silver. Direct the person to multiply the number answering to the right-hand by any even number, as 2; and that in the left by a determined odd number, as 3, and ask him whether the joint sum of the products is even or odd; or bid him half it, and so you will learn whether it is even or odd, without asking. If this sum is odd, the gold is in the right-hand; if even, *e contra*.

S P O R T XXIV.

To find two numbers, the Ratio and difference of which is given.

TO find two numbers, the first of which, for example, is the second as 5 to 2, and the difference or excess 12: multiply the difference 12 by 2, the *least* term of the given Ratio, and divide the product 24 by 3, the difference of the two terms 5, 2, and you will find the quotient 8, the least of the two numbers looked for, and that added to the difference 12, *viz.* 20, the greatest.

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If you will, you may multiply the given difference by the *greatest* term of the given Ratio, and after dividing the product by the difference of the two terms of the Ratio, you will find the quotient the great number, which, upon the subtraction of 12, leaves the lesser remaining. Or you may take this way; multiply each of the two terms of the given Ratio, by the difference given, and divide each of the products by the difference of the two terms, and the quotients are the numbers demanded. This Sport furnishes an easy solution to the following question:

Question. If a man has as many pieces of money in one hand as in the other, how shall we know how much is in each hand?

Bid him put two out of the left into the right-hand, which by that means will have 4 more than the left, and ask for the Ratio of the number of pieces in the right to that in the left, which we shall here suppose to be as 5 to 3. Then multiply 4, the difference of the two hands, by 3, the least term of the given Ratio, and divide the product 12, by 2, the difference of the two terms of the Ratio 5, 3: the quotient 6 is the number of pieces in the left, to which if you add the difference 4, you have 10 for the right. These two put together make 16, and consequently at first the man had 8 in each hand.

SPORT XXV.

Two persons having agreed to take at pleasure less numbers than a number proposed, and to continue it alternately, 'till all the numbers make together a determined number greater than the number proposed, it is required how to do it?

SUPPOSE the first is to make up 100, and both he and the second are at liberty to take alternately any number under 11; let the first take 11 from 100 as often as he can, and these numbers will remain, 1, 12, 23, 34, 45, 56, 67, 78, 89, which he is to keep in mind; and first take 1, for then let the second take what number he will (under 11) he cannot hinder the first to come at the second number 12, for if the second takes 3, for example, which, with 1 makes 4, the first has nothing to do but to take 8, and so reach 12. After that, let the second person take what number he will he cannot hinder the first from coming at the third number 23; for if he takes 1, for instance, which with 12 is 13, the first takes 10, and so makes 23. In like manner, the first cannot be hindered to reach the fourth number 34, then the fifth 45, then 56, then 67, then 78, then 89, and at last 100.

As for the second person, he can never touch at 100, if the first understands the way. Indeed if the first takes 2 at the beginning, his business is to take 18, and so clap in upon 12, with

with the same advantage the first had above. But if the first is acquainted with the artifice, he will be sure to take 1, and so the second can never make 12, nor 23, &c. nor, in fine 100.

If the first would be sure to win, he must take care that the lesser number proposed does not measure the greater; for if it does, he has no infallible rule to go by. For example, if, instead of 11, 10 were the number proposed; taking 10 continually from 100, you have these numbers, 10, 20, 30, 40, 50, 60, 70, 80, 90; now the first being obliged to pitch under 10, cannot hinder the other from making 10, and so 20, 30, &c. and in fine 100.

You need not be at the pains to make a continued subtraction of the lesser number from the greater, in order to know the numbers the first is to run upon; for if you divide the greater by the lesser, the remainder of the division is the first number you are to take. Thus divide 100 by 11, 1 is the remainder for the first number, add to that 11, it makes 12 for the second, and 12 with 11 makes 23 for the third, and so on to 100.

S P O R T XXVI.

To divide a given number into two parts, the Ratio of which is equal to that of two numbers given.

SUPPOSE 60 is to be divided into two numbers, the least of which must be to the greater

greater as 1 to 2 : add together the two terms of the given Ratio 1, 2, and divide 60 by their sum 3 : the quotient 20 is the least number wanted, and that subtracted from 60 leaves 40 the greater. Or, multiply the two terms 1, 2, separately, by 60, and divide each of the products, 60, 120, by 3, the sum of the terms ; and the two quotients, 20, 40, are the numbers you look for. This Sport gives an easy solution to the following question :

Question. To divide the value of a crown into two different species or denominations, the number of which shall be equal.

The solution being demanded in integers, it is impossible to solve this or the like Sport, unless the sum of the two terms of the Ratio of the different species proposed does exactly divide the crown when reduced to smaller money. Thus it is impossible to divide an English crown according to the tenour of the question, into shillings and pence ; because the Ratio of these species or denominations is 12, 1 : and 13, the sum of these two terms, does not exactly divide 60 pence, the value of the crown : but make the two species pence and farthings it will do ; since 4, 1, the terms of their Ratio, make together 5, which exactly divides 240, the value of the crown in farthings ; and the quotient 48, solves the question, that is, 48 pence, and 48 farthings make a crown.

S P O R T XXVII.

To find a number which being divided by given numbers separately, leaves 1 the remainder of each division; and when divided by another number given, leaves no remainder.

TO find a number which leaves 1 remaining, when divided by 5 and by 7, and nothing when divided by 3. multiply into one another the two first numbers given, 5, 7; to their product 35, add 1, which makes 36, the number demanded. For if you divide 36 by 5 and by 7, the remainder is 1; and when you divide it by 3, there is as it happens, no remainder.

After finding this first and lowest number of the proposed quality 36, you may find an infinite quantity of greater numbers of the same quality, and that in the following manner; Add the first number found 36, to 105, the product of the three given numbers 5, 7, 3, and the sum 141 is a second number of the same quality, then add to 141 the product above-mentioned 105, and you have 246 for a third; which, with the addition of 105, makes 351 for a fourth number; and so on.

To find a number that divided separately by 2, 3, 5, leaves 1 remaining, and no remainder when divided by 11: if you take 30, the product of the first three numbers 2, 3, 5, and add 1 to it, you have the number 31, which divided by each of the three first numbers, 2, 3, 5, there should remain 1, and by 11, the fourth

number, nothing: but so it is, that 31, when divided by 11, leaves 9 remaining, and therefore 31 is not the right number; but in order to find out the right number, take 30 the product of the three terms, 2, 3, 5, and quadruple it, which makes 120, which with the addition of 1, is the number required 121, and that added to 1320, the product of the four numbers given 2, 3, 5, 11, makes 1441 for a second number of the same quality; and so on, as above. In this case 30, the product of 2, 3, 5, being divided by 11, left 8 remaining, and the quadruple of that 8, 32, being but 1 short of 33, the multiple or tripple of 11, we quadrupled the 30, and added to the sum.

In like manner, to find a number that divided separately by 3, 5, 7, leaves 2 remaining, and no remainder when divided by 8: divide 105, the product of the three first numbers 3, 5, 7, by the fourth 8; and because there remains 1, multiply the product 105 by 6, that the product 630 divided by 8 may leave a remainder of 6, which is less than 8 by 2, and then adding 2 to the last product 630, you have 632 the number required, which added to the product of the four given numbers, makes a second number of the same quality; and that with the same addition, a third, and so on.

To find a number that divided separately by 3, 5, 7, leaves 2 remaining, and divided by 11, leaves no remainder: divide 105, the product of the first three numbers given, 3, 5, 7,
by

by the fourth 11; and in regard there remains 6, the double of which, 12, surpasses the divisor 11 by 1; multiply the product 105 by 2, that 210 being divided by 11, there may remain 1; and since it is desired that 9 may be the remainder, which is less than the divisor 11 by 2, multiply the last product 210 by 9, and then the product 1890 being divided by 11, the remainder will be 9; and therefore adding 2 to that last product, you will have a number 1890, which leaves no remainder, being divided by 11.

In like manner to find a number that being divided by 5, or 7, or 8, leaves 3 remaining, and nothing when divided by 11: multiply by 9, 280 the product of the first three numbers given, 5, 7, 8, and the product 2520 being divided by 11, there remains 1, upon which you may make the remainder 8, which is less than 11 by the given number 3, by multiplying the foregoing product 2520 by 8, which makes 20163, and consequently that sum, with the addition of 3, viz. 20163, is the number sought for. This Sport directs us to solve the following Question:

Question. To find how many pistoles were in a purse that a man has lost, but remembers, that when he told them by two's, by threes, or by fives, there always remained an odd one, and when he counted them by sevens, there remained none.

Here we are to find a number that, when divided by either 2, or 3, or 5, still leaves 1

remainder; and when divided by 7 leaves 0. Now there are several numbers of that quality, as appears from the foregoing Sport; and therefore to find the number that really was in the purse, it behoves us to be directed by the bulk or weight of the purse, in order to determine the real number.

Now to find the least of all these numbers, let us first of all try for a number that is exactly divisible by 2, by 3, and by 5, and likewise by 7 when 1 is added to it. If you multiply together the three first numbers given, 2, 3, 5, their product 30 will be divisible by each of these three numbers; but when you have added 1 to it, the sum 31 is not divisible by the fourth number given, 7, for there remains 3, and since the product 30, when divided by 7, leaves 2, its double 60 will leave 4 upon the like division, and by the same consequence its tripple 90 will leave 6 remaining. Now 6 wanting but one of 7, add that one to this tripple number 90, and so 91 will be exactly divisible by 7, and consequently is the number sought for.

To find the next larger number that answers the Question, multiply together the four given numbers 2, 3, 5, 7, and to their product 210 add the first and least number found 91; the sum 301 is the second number sought for; and if you add to this second number the foregoing product 210, the sum 511 will be the third number that solves the Question; and so on *in infinitum*.

Thus,

Thus, to resolve the Question, you may answer, that there might be in the purse 91 Louis d'Ors, or 301, or 511; and the bulk of the purse will serve to direct you which of the numbers was really in it.

S P O R T XXVIII.

Of several numbers given to divide each into two parts, and to find two numbers of such a quality, that when the first part of each of the given numbers is multiplied by the first number given, and the second by the second, the sum of the two products is still the same.

SUPPOSE, for example, these three numbers given, 10, 25, 30, and the solution is required in entire numbers; take any two numbers for the two numbers sought for, provided their difference be 1; or such as may exactly divide the product under the greatest of these two numbers, and the difference of any two of the three given numbers, and so, that the greatest of these two numbers multiplied by the least given number 10, may be greater than the least of these two numbers multiplied by the greatest given number 30; such are 2 and 7.

The two numbers required, 2 and 7, being thus found; the first part of the first given number 10, may be taken at pleasure, provided it is less than 10, and then the number arising from the subtraction of the least found number 2, multiplied by the greatest given number

number 30, from the greatest found number 7, multiplied by the least given number 10; and then the number arising from the division of the remainder 10 by 5, the difference of the two numbers found 2, 7; that is, less than 2, which is 1, which being subtracted from the first given number 10, leaves the remainder 9 for the other part; and that being multiplied by the second number found 7, and the first part 1 being multiplied by the first number found 2, the sum of the two products 63 and 2 is 65.

To find the first part of the second number given, 25, multiply 15, the difference of the first two numbers given, 10, 25, by the greatest number found 7; and divide the product 105 by 5, the difference of the two numbers found 2, 7; then add the quotient 21 to 1, the first part found of the first number given 10; and the sum 22 will be the first part of the second number given 25, and consequently the other part will be 3, which being multiplied by the second number found 7, and the first part 22, being multiplied by the first number given, 2, the sum of their two products 21, 44, makes likewise 65.

Last of all, to find the first part of the third number given 30, multiply 5, the difference of the two last numbers given, 25, 30, by the greatest number found, 7, and divide the product 35 by 5, the difference of the two numbers found 2, 7; then add the quotient 7 to 22, the first part of the second number given 30,

30, and the sum 29 will be the first part of the third number given 30, and consequently the other part will be 1, which being multiplied by the second number found 7, and the first part 29 being multiplied by the first number found 2, the sum of the two products 7, 58, makes still 65.

Or else multiply 20, the difference of the first and the third number given, by the greatest number found 7, and divide the product 140 by 5, the difference of the two numbers found 2, 7; then add the quotient 28 to 1, the first part of the first number given 10, and you will have 29 as above, for the first part of the third number given 30.

If you take 1 and 6 for the two numbers sought for, and 4 for the first part of the first number given 10, in which case the other part will be 6, which being multiplied by the second number found, 6, and the first part 4 by the first number found 1, the sum of the two products 36 and 4, is 40. Upon this supposition, I say, the first part of the second number given 25, will be 22, and consequently the other part 3, which being multiplied by the second number found 6, and the first part 22 by the first found number 1, the sum of the two products 18, 22, is likewise 40; and in fine, the first part of the third number given 30, will be 28, and the other 2, which being multiplied by the second number given 6, and the first 28 by the first 1, the sum of the two products is still 40.

This

This Sport directs us to the solution of the following Question:

Question. One woman sold at market 10 apples at a certain rate apiece; another sold 25 at the same rate; and a third sold 30 still at the same price; and yet each of them brought the same sum of money home with them. The question is how this could be?

It is manifest, that to save the possibility of the Question, the women must sell their apples at two different sales, and at two different rates, seeing at each sale or division they sell at the same rate. Let the two different rates be 2 and 7, which are the two numbers that we found in the foregoing Sport; and we will suppose

	<i>Apples</i>		<i>Farthings</i>		<i>Apples</i>		<i>Farth.</i>	
X.	1	at	2		9	at	7	} 65
XXV.	22	at	2		3	at	7	
XXX.	29	at	2		1	at	7	

that at the first sale they sold at 2 farthings an apple, and that at this rate the first sells 1 apple, the second 22, and the third 29; the three numbers 1, 22, 29, being the first parts of the three given numbers X, XXV, XXX, which were found in the foregoing Sport; in this case the first woman will take 2 farthings, the second 44, and the third 58. In the next place, if we suppose they sell the rest of their apples at 7 farthings, then the first woman will take 63 farthings for the 9 apples she had left, the second will take 21 farthings for the 3 apples she had left, and the third 7 farthings for the 1 apple she

she had left ; and so each of them will take in all 65 farthings.

Or, if you will, make the two different rates 1 and 6, which were the two numbers found in the last Sport ; and suppose at the first sale they sell at a farthing an apple, at which price the first sells 4, the second 22,

	<i>Apples</i>		<i>Farthings</i>		<i>Apples</i>		<i>Farth.</i>	
X.	4	at	1		6	at	6	} 40
XXV.	22	at	1		3	at	6	
XXX.	28	at	1		2	at	6	

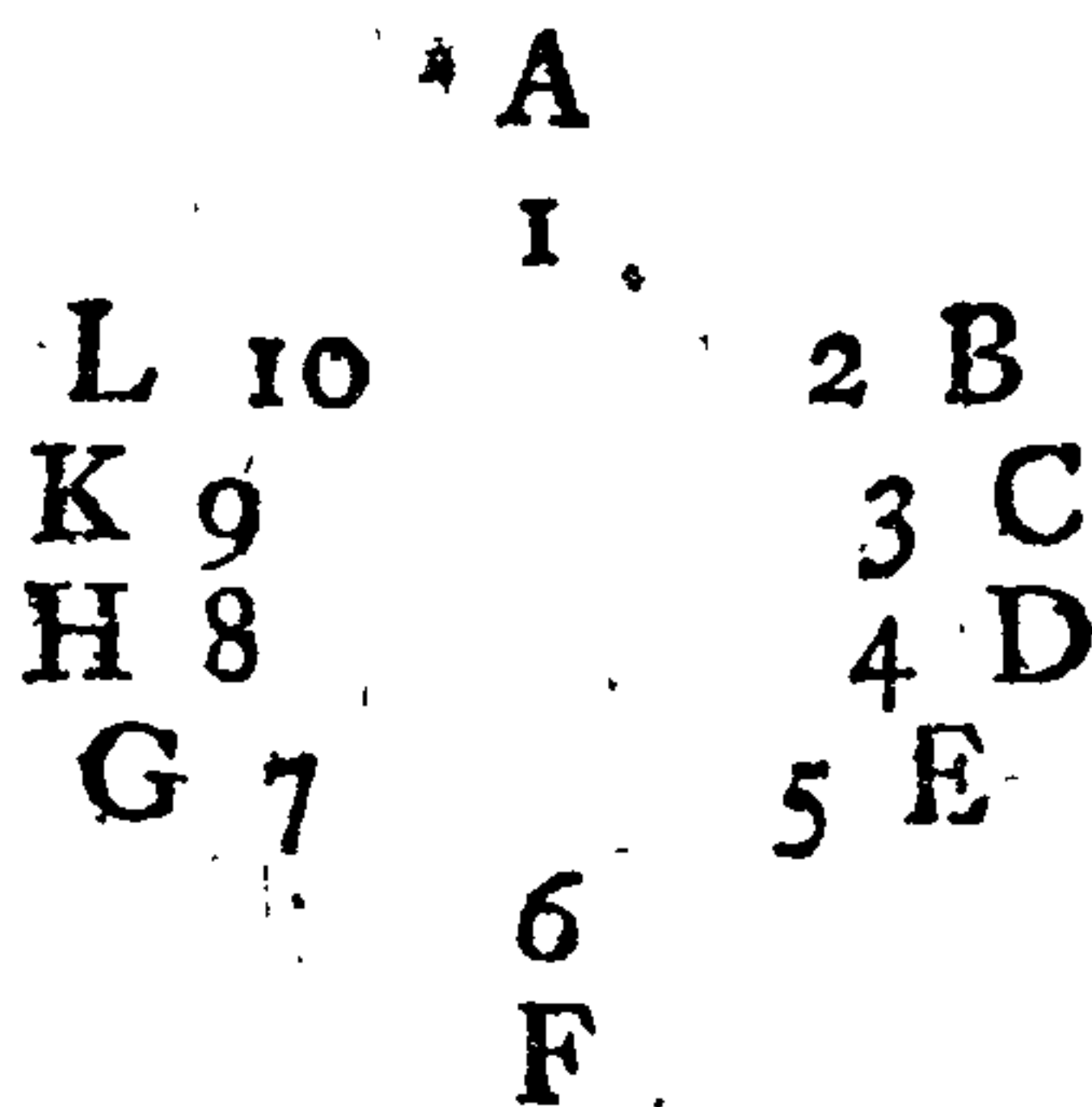
and the third 28 ; these three numbers 4, 22, 28, being the first parts of the given numbers X, XXV, XXX, which were found in the last Sport ; the first woman will take 4 farthings, the second 22, and the third 28. Then suppose again that they sell the rest of their apples at 6 farthings a piece, the first woman will take 36 farthings for the 6 apples she had left, the second 18 for the 3 apples she had left, and the third 12 farthings for the 2 apples she had left. And thus every one of them will take in all 40 farthings.

S P O R T XXIX.

Many numbers which proceed from 1 or unity, in a progression, according to the natural order of numbers, (such as these, 1, 2, 3, 4, 5, 6, &c.) being placed in a round form like a ring ; to discover which of these numbers any one has thought upon.

LET

LET any multitude of numbers in the
aforesaid progression, suppose these ten, to
wit, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, be marked
upon ten ivory-counters (or for want thereof
upon ten small pieces of paper) which may be
represented by these ten letters, A, B, C, D,
E, F, G, H, K, L, viz. suppose 1 to be writ
upon the counter A, 2 upon B, 3 upon C, &c.
Then having placed those counters circularly
as you see (with their blank faces uppermost,
and the figures underneath, that the subtilty



of the Sport may the better be concealed) let
any one think upon any number of units which
does not exceed ten; that done, bid him touch
one of those counters at pleasure, and to the
number on the back side of the counter touched
(which you cannot be ignorant of, having no-
ted well the place of 1 or A) add secretly in
your mind, the just number of all the counters
and reserve the sum; then bid him imagine in
his mind the counter touched to be the number
which he thought, and from that counter to
count backwards, until he has made up the
aforesaid sum, which you reserved, so will his
com-

computation infallibly end on the counter upon which the number thought of is marked.

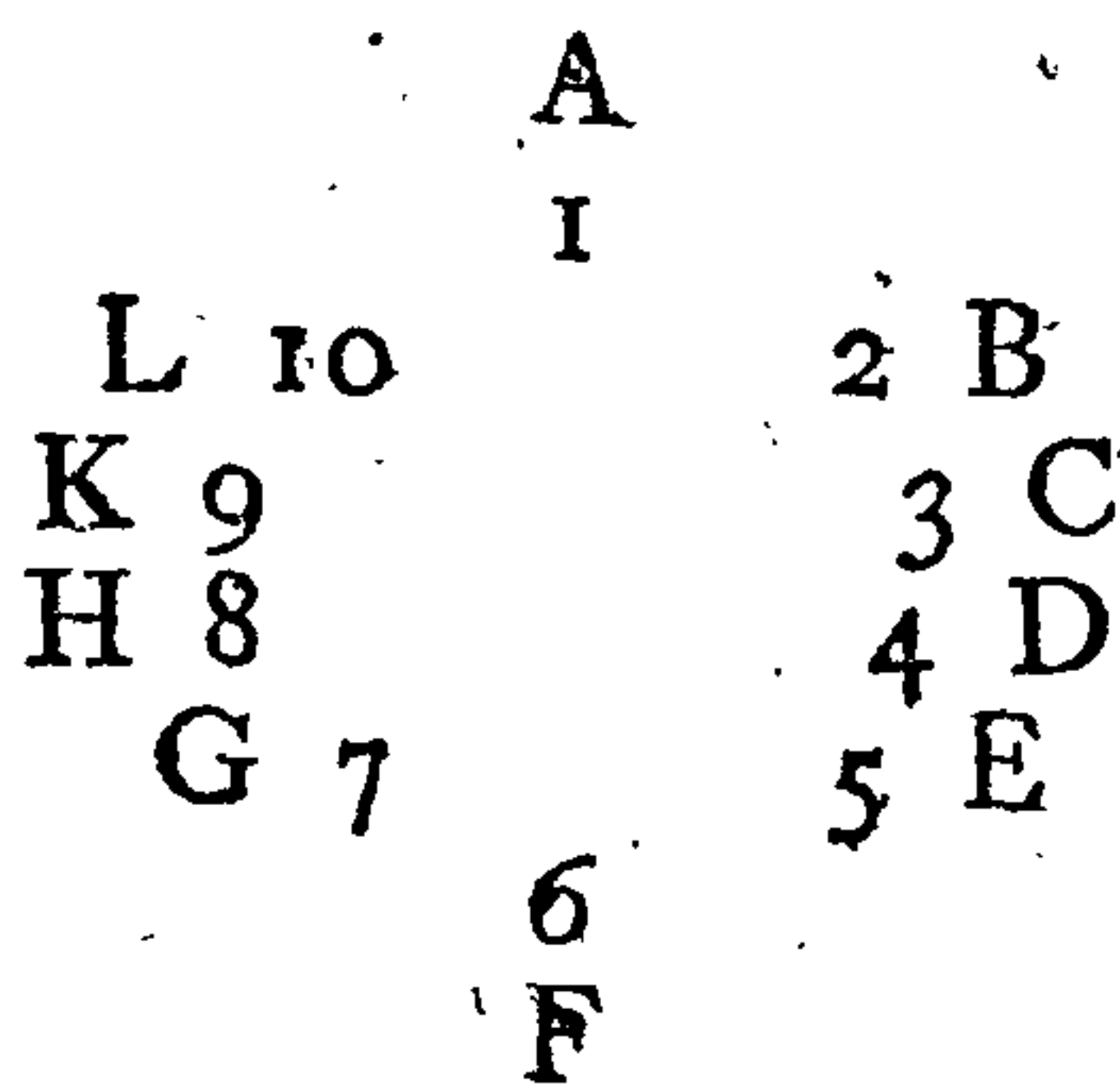
For example, suppose he thought 7 or G, and that he touched B, to wit, 2; add to 2 the number of all the counters, to wit, 10, so the sum will be 12; then bid him count to 12, beginning at B and going backward, and esteeming B to be the number thought, to wit, 7; so will 8 fall upon A, 9 upon L, 10 upon K, 11 upon H; and lastly, 12 upon the counter G, which being turned up will shew the number thought.

The reason of this rule is not difficult to be apprehended, two principles being pre-supposed the one is this; to wit, many counters or things whatsoever being disposed orderly one after the other, in one continued line, whether it be right or circular; if you value or name the first counter to be some number of units at pleasure, and continue the count forward according to the natural order of numbers, until another number be named which falls upon the last counter: Or, if you imagine or name the last counter, to be the same number of units as before you put upon the first, and continue to count backwards to the first counter; I say that the same number will be named at the end of both these computations: For example, in these 9 letters, A, B, C, D, E, F, G, H, K, if the letter A be esteemed to be 4, and from thence you count forwards unto K, according to the natural order of numbers, the letter K will fall upon the number 12. In like manner, if you esteem K
to

to be 4, and count backwards from K to A, the letter A will likewise fall upon 12.

4.	5.	6.	7.	8.	9.	10.	11.	12.
A.	B.	C.	D.	E.	F.	G.	H.	K.
12.	11.	10.	9.	8.	7.	6.	5.	4.

The other principle is this, to wit, many counters being disposed in a round form like a ring; if you esteem any one of those counters to be some number at pleasure, and then from that counter if you count circularly, until you end upon the counter where you began, the number last named will be equal to the sum of the number of all the counters, and of the number which you put upon the first counter; for example, if D be one of the ten letters placed in a circumference, and that imagining D to be 7, you begin with it, and count round the whole circumference, according to the natural progression of numbers till you end with



D where you began; the number 17, which is composed of 10 and 7, will necessarily fall upon D; for 9 (which is the number of letters in the circumference besides D) being added to

to 7 (which was first put upon D) makes 16, to which 1 being added, (because D ends as well as begins the circumference) the sum is 17.

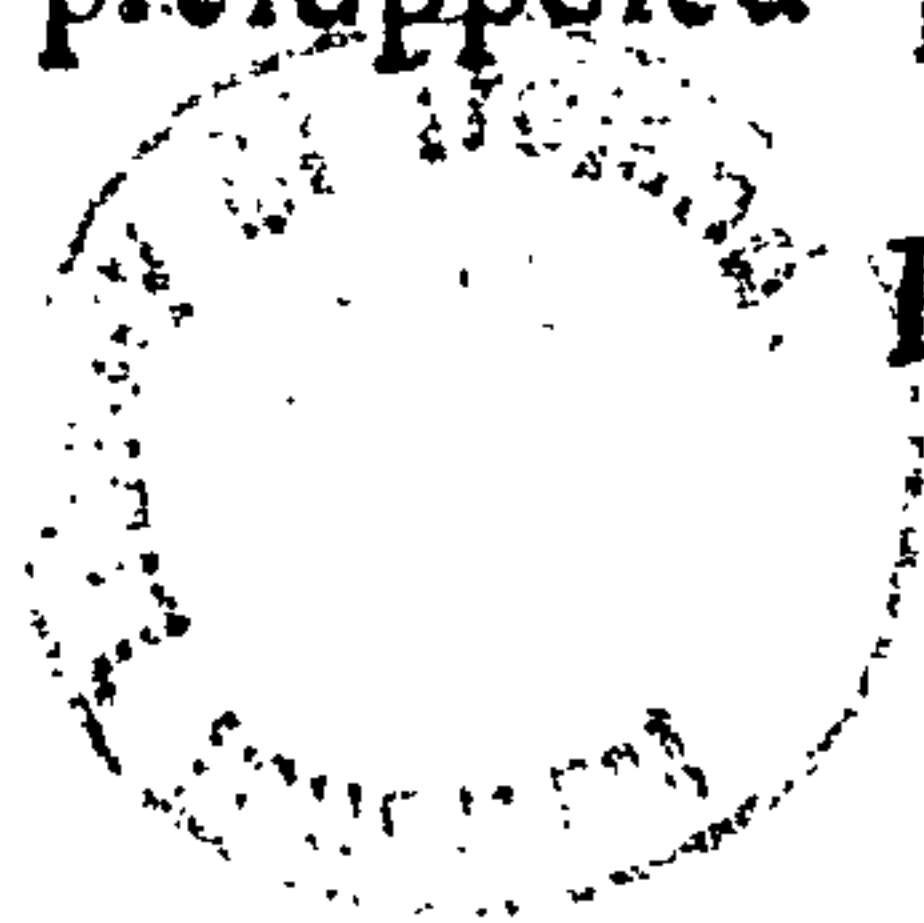
Now these two principles being pre-supposed, it will not be difficult to apprehend the reason of the aforesaid Rule in all cases that can happen; for imagine that one has thought upon 7, or the counter G, then that counter which he shall touch must either be the same counter G, or some other that precedes or follows G.

First therefore, supposing the counter or number touched to be the same with the number thought, the truth of the Rule will be then evident; for by the Rule given, he will begin to count from the same G to 17, putting 7 upon G, therefore by the second presupposition the number 17 will fall upon G.

Secondly, imagine that he touched a counter or number following G the number thought, as L or 10; then according to the Rule adding 10 (the numbers of all the counters placed circularly) to 10 or L, the (counter touched (bid him count backwards to 20 by beginning at L, and esteem L to be 7. Now, because by beginning to count at G which is 7, and proceeding to count forward, the number 10 will fall upon L; therefore by the first presupposed principle, if we esteem L to be 7, and count backwards, the number 10 will infallibly fall upon G, and then the number 20 shall also fall upon the same G by the second presupposed principle.

H

Lastly,



Lastly, imagine he touched some number or counter which precedes 7 the number sought, as B or 2; then adding 10 to 2, you are to bid him count unto 12, he having first imagined B to be the number thought 7; and going backward to A, L, K, &c. Now because by proceeding to count at B, which is 2, and beginning to count forward to C, D, &c. the number 7 falls upon G; therefore if one imagine that G is 2, and from thence count backwards towards E, F, &c. the number 7 will fall upon B (by the first presupposed principle) therefore when one assumes B to be 7, and counts towards A, L, &c. to any assigned number, it is in effect as much as when one imagines G to be 2, and counts towards E, F, unto the said assigned number, for each of those computations will end in the same point; but it is manifest (by the second presupposed principle) that esteeming G to be 2, and counting towards E, F, D, &c. round the whole circumference; the number 12 will fall upon the same G. And because G being supposed to be 2, and counting on the same coast as before, the number 7 falls upon B; therefore if the computation be continued on the same coast as before, the number 7 falls upon B; therefore if the computation be continued on the same coast from B 7, to 12, the number 12 will fall upon the same G. So that the practice of this Sport in all its cases is demonstrated.

Note, That to the number of the counter touched, you may not only add the number
of

of all the counters once (as the Rule directs) but twice, thrice or more times: For example, B being touched, you may cause him to count to 12, or 22; or to 32, 42, &c. the reason whereof is evident from the second presupposed principle.

S P O R T XXX.

Many numbers being shewed by pairs, to wit, two by two unto any person, that he may think upon any one of those pairs at pleasure, to discover the pair that was thought upon.

L E T twenty numbers, suppose these, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, be writ upon ivory-counters (or for want thereof upon small pieces of paper) to wit, 1 upon one counter, 2 upon another, 3 upon a third, &c. Then dispose them into pairs, as you see, viz. suppose 1 and 2 to be one pair, 3 and 4 to be another pair, &c. and of these pairs let any one think upon which pair he pleases. That done, you are to distribute the said 20 numbers into ranks, in form of a long square, until there be 5 numbers in length, and 4 in breadth, after this manner, viz. lay the three first numbers, 1, 2, and 3, in a rank (as you see in the second figure) from A towards B; then place 4 under 1, and 5 after 3, (in the said rank A, B). Again, place 6 under 4, and 7 after 5, (in the said rank A, B). Then place 8 under 6, also 9, 10, 11,
H 2 on

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20

on the right-hand of 4, in the rank C, D. Again, place 12 under 9, and 13 on the right-hand of 11, in the rank C, D, and 14 under 12. Moreover, placing 15, 16, 17, on the right-hand of 12, in the rank E, F. Lastly, place 18, 19, 20, on the right-hand of 14, in the rank G, H, so will all the numbers be ranked as you see in the Table. That done, you are to demand of him that thought upon two numbers as aforefaid, in what rank or ranks the faid numbers happen to be found, *viz.* in which
of

of the ranks A B, C D, E F, G H, or in which two of the said ranks: Now if he answer

A	1	2	3	5	7	B
C	4	9	10	11	13	D
E	6	12	15	16	17	F
G	8	14	18	19	20	H

that the two numbers he first thought upon are in the first rank A, B, then 1 and 2 will be the numbers thought or kept in mind; if in the second C D, then 9 and 10 shall be the numbers thought of; if in the third rank E F, then 15 and 16 will be the numbers thought: If they are in the fourth rank G H, then 19 and 20 shall be the numbers thought; but if he say that the numbers thought are in different ranks, then you are heedfully to mark the said numbers 1, 2, 9 and 10, 15 and 16, 19 and 20, which may be called the keys of the sport, in regard they serve not only to discover the two numbers thought, when they are both in one and the same rank (as aforesaid) but even when they are in two different ranks: For in this latter case, as soon as it hath been declared to you in which two ranks the two numbers thought are placed, you are to take the key of the highest of those two ranks, and descending in a down right line from the first number of that key to the lower of the said two ranks, you'll

H 3

there

there find one of the two numbers thought, and upon the right-hand of the second number of the said key, at the same distance sideways, from the second number of the key, (as one of the numbers thought was distant from the first number of the key) you will find the other number thought.

For example, suppose the two numbers thought are 7 and 8, and it is declared to you, that they are in the first and fourth ranks; take then the key of the highest of those two ranks, to wit, of the first, which is 1 and 2, and descending downright from 1 to the fourth rank, you'll there find 8 one of the numbers thought: then seek sideways on the right-hand of 2 (the second number of the key) a number as far separated from 2, as 8 is distant from 1, and you'll find 7 the other number thought.

Again, suppose he says that the numbers thought are in the second and third ranks: Take then the key of the second rank which is 9 and 10, and descending downright from 9 to the third rank, you shall there find 12, which is one of the numbers thought; then seek sideways on the right-hand of 10, (the second number of the key) a number as far distant from 10, as 12 is from 9, and you'll find 11, which is the other number thought.

The reason of this will be apparent, from a serious consideration of the placing of the numbers according to the Rules before given: for it is thereby evident, that of the first numbers coupled two by two, there can never be found
more

more than one pair in one and the same rank ; and of all the other pairs, one number is always found in one rank, and the other number in another rank.

Note also, that this Sport may be practised with divers persons at once, and not only with 20 numbers, but with any such multitude of numbers as is produced by the multiplication of any two numbers that differ by 1, or unity ; as 30, which is the product of 5 multiplied by 6, and 42, which is the product of the multiplication of 6 and 7. That which is chiefly to be regarded is, the placing of the numbers in ranks according to the directions before given : And for the more easy comprehending of that order, I have, in the following Table, ranked 30 numbers in their due places, which being compared with the former Table, and well viewed, will be a clearer illustration than can be expressed by many words.

1	2	3	5	7	9
4	11	12	13	15	17
6	14	19	20	21	23
8	16	22	25	26	27
10	18	24	28	29	30

SPORT XXXI.

Among three persons, to find how many cards or counters each of them has got.

LET the third person take what number of cards, or counters he pleases, provided it be evenly even, that is, divisible by 4; let the second take as many 7's as the other has taken 4's; and the first as many 13's. Then bid the first give to the other two as many of his counters as each of them had before; and the second to give to the remaining two as many of his counters as each of them; and in like manner, the third to give to each of the other the same number that they have. By this means it will so fall out, that they will all have the same number of counters, and each of them will have double the number that the third had at first. And for this reason, if you ask one of the three how many counters he has got, half his number is the number the third had at first; and if you take as many 7's, and as many 14's as there were 4's in the third person's number, you will have the number of cards or counters that the second and first took.

For example, if the third took 8 cards, it behoved the second to take 14, that is twice 7, because there is twice 4 in 8;

1st 2d. 3d. and the first must take 26, that is twice 13 by the same reason.

26	14	8	If the first, who has 26 cards,
4	28	16	gives to the second 14, that is,
8	8	32	as many as he had at first: and
16	16	16	to the third 8, that being his first
			number,

number, he will have only 4 left to himself; and the second will have 28; and the third 16. But if the second, who had 28 cards, gives out of his cards 4 to the first, who had just as many before, and 16 to the third, who had likewise as many; he will have 8 left to himself, and the first will have 8, and the third 32. In fine, if the third, who has got 32, gives 8 to each of the others, all the three will have 16, which is the double of 8, the number that the third took up at first.

S P O R T XXXII.

Of three unknown cards, to find what card each of three persons has taken up.

THE number of each card taken up must not exceed 9. Then, to find out that number, bid the first subtract 1 from double the number of the points of his card, and after multiplying the remainder by 5, add to the product the number of the points of the second persons card. Then cause him to add to that sum 5, in order to have a second sum; and after he has taken 1 from the double of that second sum, make him to multiply the remainder by 5, and add to the product the number of the points of the third person's card. Then ask him the sum arising from this last addition; for if you add 5 to it, you will have another sum composed of three figures, the first of which towards the left is the number of the points of the card that the first

first person took up; the middle figure will be that of the second person's card; and the last towards the right, directs you to the third person's card.

For example; if the first took a 3, the second a 4, and the third a 7; by taking 1 from 6, the double of the first 3, and multiplying the remainder 5 by 5, we have 25 the product, to which we add 4, the number of the second person's card, which makes 29, and that with the addition of 5, makes the second sum 34, the double of which is 68, and taking 1 from that, there remains 67, which being multiplied by 5, makes 335, and this by the addition of 7, the number of the third person's card, and 5 over and above, makes the last sum 347, the three figures of which severally represent the number of each card.

Another way, Or, if you will, you may bid the first add 1 to the double of the number of the points of his card, and multiply the sum by 5, and add to the product the number of the second person's card. Then bid him add in like manner 1 to the double of the preceding sum, and multiply the whole by 5, and add to the product the number of the third person's card. Then ask him the sum arising from the last addition, and subtract 55 from it, that so there may remain a number composed of three figures, each of which represents, as above, the number of each card.

As in the foregoing example, by adding 1 to 6, the double of 3, the number of the first person's

son's card, and by multiplying the sum 7 by 5, we have 35, which, with the addition of 4, the number of the second person's card, makes 39, the double of which is 78, to which if we add 1, and multiply the sum 79 by 5, we have 395; to that we add 7, the number of the third person's card, and so have 402, from which if we subtract 55, the remainder is 347, the three figures of which severally represent the number of each card.

S P O R T XXXIII.

Of three cards known, to find which and which is taken up by each of three persons.

OF the three known cards, we shall call one A, the other B, and the third C, and leave each of the three persons to pitch upon 1 of the three, which may be done six different

1st.	2d.	3d.		ways, as you see in the annexed scheme.
12	24	36	<i>Sums.</i>	Give the person the
A	B	C	23	number 12, the second
A	C	B	24	24, the third 36.—
B	A	C	25	Then direct the first
C	A	B	27	person to add together
B	C	A	28	the half of the num-
C	B	A	29	ber of that person that
				has taken the card A,

the third part of the number of the person that takes the card B, and the fourth part of the number of the person that takes the card C; and

and then ask him the sum, which you will find to be either 23 or 24, or 25, or 27, or 28, or 29, as you see in the table or scheme which shews, that if the sum is, for example, 25, the first will have taken the card B, the second the card A, and the third the card C; and if the sum is 28, the first has taken the card B, the second the card C, and the third the card A; and so on in the other cases.

S P O R T XXXIV.

To find out among several cards, one that another has thought of.

HAVING taken out of a pack of cards a certain number of cards at pleasure, and shewn them in order upon the table, before the person that is to think, beginning with the lowermost, and laying them cleverly one above another, with their figures and points upwards, and counting them readily, that you may find out the number; which, for example, we shall here suppose to be 12; bid him keep in mind the number that expresses the order of the card he has thought of, namely 1, if he has thought of the first, 2, if he has thought of the second, 3, if he has thought of the third, &c. Then lay your cards one after another, upon the rest of the pack, in a contrary situation, and putting that upon the pack first that was shewn upon the table, and that last that was last shewn. Then ask the number of the card thought of, which
we

we shall here suppose to be 4, that is, the fourth card in order of laying down, is the card thought of. Lay your cards, with their faces up, upon the table, one after another, beginning with the uppermost, which you are to reckon 4, the number of the card thought of; so the second next to it will be 5, and the third under that 6, and so on till you come to 12, the number of the cards you first pitched upon to shew the person; and you will find the card that the number 12 falls to, to be the card thought of.

S P O R T XXXV.

Several parcels of cards being proposed or shewn, to as many different persons, to the end that each person may think upon one, and keep it in his mind; to guess the respective card that each person has thought of.

WE will suppose there are 3 persons, and 3 cards shewn to the first person, that he may think upon one of them, and these three cards laid aside by themselves; then 3 other cards held before the second person, for the same end, and laid apart; and at last, 3 different cards again to the third person, for the same end, and likewise laid apart. This done, turn up the 3 first cards, laying them in three stations; upon these three lay the next three other cards that were shewn to the second person; and above these again the least three cards. Thus

Thus you have your cards in three parcels, each of which consists of 3 cards. Then ask each person in what list is the card he thought of; after which it will be easy to distinguish it; for the first person's card will be the first of his heap; and in like manner the second's will be the second in his; and the third person's card will be the third in his.

S P O R T XXXVI.

Several cards being sorted into three equal heaps, to guess the card that one thinks.

IT is evident that the number of cards must be divisible by 3, since the three lists are equal. Suppose then there are 36 cards, by consequence there are 12 in each list: ask in what list is the card thought upon; then put all the heaps together, so as to put that which contained the card thought upon between the other two; then deal off the 36 cards again into three equal hands, observing that order of the first card to be first, the second to the second, the third to the third, the fourth to the first again, and so round, dealing one card at a time, till the cards are dealt off. Then ask again, in what hand or heap is the card thought upon, and after laying together the cards, so as to put that list which contained the card between the other two, deal off again, as you did before, into three equal lists. Thus done, ask once more, what list the card is in, and you will easily

sily distinguish which is it, for it lies in the middle of the list to which it belongs; that is, in this example, it is the sixth card; or, if you will, to cover the artifice the better, you may lay them all together, as before, and the card will be in the middle of the whole, that is, the eighteenth.

S P O R T XXXVII.

To guess the number of a card drawn out of a complete stock.

AFTER one hath drawn what card he pleases out of a complete stock of 52 cards, for instance, such as we play at *Ombre* with, you may know how many points are in the card thus drawn, by reckoning every faced card 10, and the rest according to the number of their points; then looking upon the rest of the cards one after another, add the points of the first card to the points of the second, and the sum to the points of the third, and so on, till you come to the last card, taking care all along to cast out 10, when the number exceeds it; upon which account you see it is needless to reckon in the 10's or the faced cards, since they are to be cast out however. Then if you subtract your last sum from 10, the remainder is the number of the drops of the card drawn.

It is easy to know that when nothing remains the card drawn is either a 10 or a faced card: and that in this case, if it be a faced card, one cannot

cannot distinguish whether it be a king, queen or knave: now, in order to be master of that distinction, the best way is to make use of a stock of 36 cards only, such as we formerly used for Piquet, and reckon a knave 2, a queen 3, and a king 4.

If you make use of a stock of 32 cards only, such as is now used for Piquet, you are to follow the same course as is above prescribed, only you must always add 4 to the last sum, in order to have another sum, which being subtracted from 10 if it be less, or from 20, if it surpasses 10, the remainder will be the number of the card drawn; so that if 2 remains it is a knave, if 3 a queen, if 4 a king, &c.

If the stock is not full, you must take notice what cards are wanting, and add to the last sum the number of all the cards that are wanting, after subtracting from that number as many 10's as are to be had; upon which, the sum arising from this addition is to be subtracted, as above, from 10 or from 20, according as it is above or under 10. This done, it is evident by casting your eye once more upon the cards, you may tell what card was drawn.

S P O R T XXXVIII.

To guess the number of the points or drops of two cards drawn out of a complete stock of cards.

LET a man draw at pleasure two cards out of a stock of 52 cards; bid him add to
each

each of the cards drawn as many other cards as his number is under 25, which is the half of all the cards, wanting 1, fixing upon each faced card what number he pleases; as if the first card be 10, add to it 15 cards; and if the second card be 7, add to it 18 cards; so that in this example there will remain but 17 cards in the stock, the whole number taken out amounting to 35. Then taking the remainder of the pack into your hands, and finding they are but 17, conclude that 17 is the joint number of all the points of the two cards drawn.

To cover the artifice the better, you need not touch the cards, but order the drawer to subtract the number of the points of each of the two drawn cards from 26, which is half the number of all the cards, and direct him to add together the two remainders, and acquaint you with the sum, to the end you may subtract it from the number of the whole stock, *i. e.* 52; for the remainder of that subtraction is what you look for.

For *Example*. Suppose a 10 and a 7 are the cards drawn; take 10 from 26, there remains 16; and taking 7 from 26, the remainder is 19: the addition of the two remainders 16, 19, makes a sum of 35, which subtracted from 52, leaves 17 for the number of the drops of the two cards drawn.

The same is the management in a flock of 36 or 32 cards; only to colour the trick the better, instead of 26, the half of the cards, when they make 52, take another lesser number, but greater.

greater than 10, as 24, from which taking 10 and 7, there remains 14 and 17, the sum of which, 31, being subtracted from 52, the sum of all the cards, leaves 21 the remainder; from which subtract again 4, which is the double of the excess of 26 above 24, and so the remainder is 17, the number of the points of the two cards drawn, *viz.* 10 and 7.

If you make use of a Piquet-stock, consisting of 36 cards, instead of 18, the half of 36, the number of all the cards, take in like manner a lesser number, such as 16, from which take 10 and 7, and there remains 6 and 9, the sum of which, 15, being subtracted from 36, the number of all the cards, leaves 21 remaining; from which subtract again 4, the double of the excess of 18 above 16, and so the 17 remaining is the number of the points of the two cards drawn.

In like manner, if this Piquet stock consists only of 32 cards, instead of 16, the half of 32, the number of the whole stock, take any lesser number you will, provided it be greater than 10, such as 14, from which take 10 and 7, and the remainders are 4 and 7, the sum of which, 11, being taken from 32, leaves 21, and taking from that 4, the double of the excess of 16 above 14, you have 17 remaining, the number of the points of the 10 and the 7 drawn.

S P O R T XXXIX.

To guess the number of all the drops of three cards drawn at pleasure, out of a complete stock of cards.

TO solve this Sport as the former, after the shortest way, the number of cards contained in the stock must be divisible by 3; so that neither a stock of 52, nor one of 32, are proper; but one of 36 is, in regard 36, the number of all the cards, has 12 for its third part, which will assist us in the solution of the question, as follows:—

Let a man draw at pleasure three cards out of a Piquet stock of 36 cards; bid him add to each of these cards as many other cards as the number of their points falls short of 11, which is the third part of the number of all the cards, wanting one, allotting, as in the foregoing Sport, to each sacred card what number he pleases: as if the first card is 9, he adds to it 2 cards; if the second is 7, he adds to it 4; and if the third is 6, he adds 5; which make in all 14 cards; so that in this example the remainder of the whole stock is 22 cards, which denotes the number of all the points of the three cards drawn.

The better to colour the artifice, you need not touch the card, but bid him subtract the number of the points of each of the three drawn cards from 12, the third part of 36, the number of the whole stock, and add together

I 2

the

the three remainders, and tell you the additional sum, which you are to subtract from 36, and the remainder of that subtraction is what you look for.

As in this Example—Suppose he drew a 9, a 7 and a 6, take 9 from 12, there remains 3; take 7 from 12, there remains 5; and take 6 from 12, there remains 6; add the three remainders, 3, 5, 6, the sum is 14, which taken from 36, leaves 22 for the number of the drops of the three cards drawn.

To colour the trick the better, and to apply the rule to a stock that consists of fewer or more than 36 cards, such as one of 52 cards, make use of a number greater than 10, and lesser than 17, the third part of 52; for instance 15: bid him who drew the three cards add to each of his drawn cards as many other cards as the number of their respective points is under 15: for example, if the first card be 9, he adds to it 6 cards; if the second is 7, he adds 8; if the third is 6, he adds 9; which makes in all 26 cards; so that in this example there will remain in the main stock 26 cards. Taking the main stock into your hands, and finding you have 26 cards, subtract from 26 the number 4, which is the excess of 52, the number of the whole stock, above the tripple of $15 \div 3$, *i. e.* 48; and the remainder 22 is the number of all the points of the three cards drawn.

Or else you need not touch the cards, but bid the person that draws subtract the number of the drops of each of the three cards drawn from

16, which is 1 more than the first number 15, and add together all the remainders, and acquaint you with the sum; then do you subtract that sum from the number above-mentioned, 48, and you will find the remainder to be the number of all the points of the three cards drawn.

For example—Suppose he drew a 9, a 7, and a 6; take 9 from 16, there remains 7; take 7 from 16, there remains 9; take 6 from 16, there remains 10; add these three remainders, 7, 9, 10, the sum is 26, which subtracted from 48, leaves 22 for the number of the points of the three cards drawn.

In like manner, in a pack of 36 cards, take a larger number than 10, for instance 15; and taking notice of the additional cards, which amount to 26, as you seen but now, subtract that number, 26 from 36, the number of the whole pack, and to the remainder 10 add 12, which is the excess of the tripple of $15 \div 3$, *i. e.* 48, above 36, the number of the whole; and you will find the sum 22, to be the number of points enquired after. In a Piquet-pack or 32 cards, instead of 12 you must add 16, by reason that 16 is the remainder of 32 subtracted from 48.

In imitation of this and the foregoing Sport it will be easy to solve the question upon four or more cards drawn.

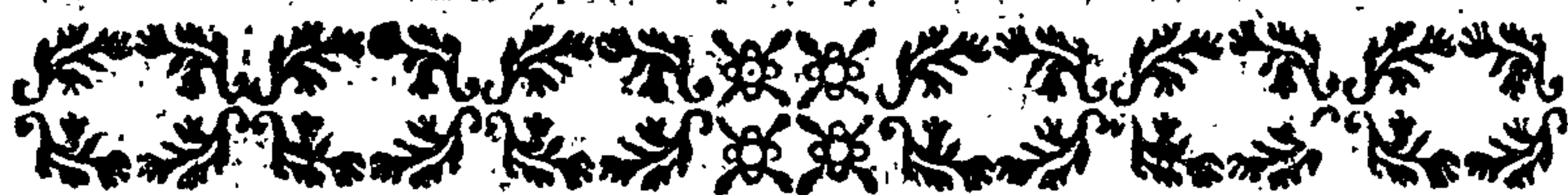
SPORT XL.

The Game of the Ring.

THIS is an agreeable game in a company of several persons, not exceeding 9 (unless you have a mind to it), in order to the easier application of the 22d Sport, *viz.* by reckoning the first person 1, the second 2, the third 3, and so on; and in like manner, reckoning the right-hand 1, the left-hand 2; the thumb of the hand 1, the fore-finger 2, the third finger 3, the fourth 4, and the little one 5; the first joint 1, the second 2, and the third 3. For if you put the ring to one in the company, for instance, the fifth person, and that upon the first joint of the fourth finger of the left hand, it is evident, that in order to guess who has the ring, and upon which hand, which finger, and which joint, one has only these four numbers to guess, 5, 1, 4, 2, the first number 5 representing the fifth person; the second 1, the first joint; the third 4, the fourth finger; and the last 2, the left-hand. Now this is performed by observing the last method of Sport 22 foregoing, as appears from the following operation.

Taking 1 from 10, the double of the first number 5, and multiplying 9, the remainder by 5, you have 45; adding to that the second number 1, you have 46, to which if you add 5, you have 51 for a second sum: the double of this second sum is 102, from which take 1, there remains 101, which being multiplied by 5, makes

5, makes 505, and that with the addition of 4, the third number, makes 509, to which if you add 5, you have this second sum 514, the double of this 1028 lessened by 1, and the remainder multiplied by 5 makes 5135, to which adding the fourth number 2, you have this sum 5137, and that augmented by 5 gives this second sum 5142, the four figures of which represent the four numbers inquired for, and by consequence denote that the ring is upon the first joint of the fourth finger of the left-hand of the fifth person.



PART II.

Artificial FIRE-WORKS.

SPORT XLI.

To make Silent Powder, or such as may be discharged without a noise.

THIS unsounding powder, if any such there is, goes commonly under the name of *White-powder*, because, possibly, the first made was of that colour. It is not probable it can be of any great force, for as much as the noise of gun-powder proceeds from the violent percussion of the air, occasioned by the strength of it. I have not indeed seen this powder myself, yet I have read in authors several ways of making the same, of which the following two only occur to my memory.

The first is thus: To one pound of common Gun-powder take half as much Venetian Borax, which having pulverised, mixed, and well incorporated together, reduce the mixture into grains, and you have the powder required.

The

The other way is—To four pounds of common Gun-powder, add two pounds of Venetian Borax, one pound of Lapis Calaminaris, and one pound of Sal armoniac; pulverise them all together to make of them a powder in grains, as before.

S P O R T XLII.

To prepare an Oil of Sulphur required in Fire-works.

HAVING melted what quantity of Sulphur you think fit, upon a moderate fire in an earthen or copper vessel, throw into it some old, or, in defect of this, some new brick that is well burned, and was never wetted, broken into many small pieces about the bigness of a bean; stir them continually with a stick, till they have drunk up and consumed all the sulphur; this done, set them upon a furnace to distil in an Alembic; so you shall have a very inflammable oil, fit for your purpose.

You may make it otherways thus—Fill one third or fourth part of a glass bottle with a long neck with Sulphur pulverised; then pouring upon it spirit of Turpentine, or oil of walnuts, or of Juniper, till the bottle is half full, set it upon hot cinders, leaving it there eight or nine hours, and you shall find an oil therein of the abovesaid quality.

S P O R T

S P O R T XLIII.

To prepare the Oil of Saltpetre useful in Fire-works.

PUT upon a fire-board, well plained and dry, what quantity of purified Saltpetre you please, and cause it to melt by putting thereupon burning coals; and you shall see the liquor to pass through the board, and to fall down drop by drop, which must be received in an earthen or copper pot, where you have an oil of Saltpetre, fit to be used in fire-works, as we shall declare in its proper place.

S P O R T XLIV.

To prepare the Oil of Sulphur and Saltpetre mixed together.

HAVING mixed and well incorporated equal portions of Sulphur and Saltpetre, reduce all into a fine powder, which must be passed through a fine scarce; put this powder thus scarced into a new earthen pot, or one that hath not been used, and pour upon it good white-wine vinegar, or else brandy, till it is covered. Then cover your pot so that no air may get into it, and set it to stand in some hot place, till all the vinegar is consumed or disappears. Last of all, draw from the remaining matter the oil by means of an Alembic; which will serve to several purposes of Fire-works.

S P O R T

S P O R T XLV.

*To make Moulds, Rowlers, and Rammers for
Rockets of all sorts.*

A Rocket, which the French call *Fusée*, the Latins *Rocbetta*, and the Greeks *Byrobolus*; consists of a Cartouch or paper tube called the Coffin, and a combustible composition, with which it is loaded; which being fired, mounts into the air, in a manner most agreeable to behold.

There are three sorts of them; the Small, the Middling and the Great. All such are reckoned Small, whereof the diameters do not exceed that of a lead bullet of one pound, or whose moulds admit not a bullet above that weight. The Middling are those the moulds of which will admit bullets from one to three pounds weight. The Great will carry from a three pound to an hundred pound ball.

To determine the bigness of these Coffins to a required measure, that is, length and thickness, and to make any demanded number of them, of the same reach, and of equal force, they must be fitted to a concave cylinder, made of some hard matter, and turned exactly in a lathe: this is called the Mould or Form, which is sometimes made of metal, but most commonly of hard wood, such as Box, Juniper, Ash, Cypress, wild Plum-tree, Italian Walnut-tree, and such like.

Besides

Besides this, there is another but a convex and solid cylinder of wood required, called a *Rowler*, upon which the thick paper, whereof the *Coffin* is made, must be rowled, till it is of a bigness exactly to fill the concavity of the *Mould*. The diameter of this *Rowler* must contain five eighth parts of that of the *Mould*, the length of which must be six times the diameter of its bore, in small *Rockets*; but in the middling and the large ones it must be only five, or four times the length of the diameter of their bore.

Another cylinder of wood must also be had, which is to be a little smaller than the former, that it may go into the *Coffin* with the greater ease. And this is to serve for a *Rammer*, to drive down the Composition into the *Coffin* when you charge it. But first your *Coffin* must be straitened or choaked, which is done by winding a cord about the end of it, after you have a little withdrawn the *Rowler*, turning in the mean time the *Coffin*, and drawing the cord, till there remains only a little hole, which then must be tied with strong packthread. This done, you must draw out the *Rowler*, and introducing the *Rammer* into the *Coffin*, put all into the *Mould*; and when you have struck five or six blows with a mallet upon the *Rammer*, to give a good form to the neck of the *Rocket*, the *Coffin* is finished, and ready to be filled upon occasion.

This *Rammer* must be bored lengthwise to some depth, that it may receive into its concavity

vity the Needle which must be in the mould, together with the Coffin and Rammer, the use of this needle, which must be one third part of the length of the Coffin or Mould is to make a vent for the priming in the bottom of the Composition, of which we speak in the ensuing Sport.

S P O R T XLVI.

To prepare a Composition for Rockets of any size.

TH E Composition wherewith the coffins are to be filled is different according to the different bigness of them; for it is found by experience, that what is fit for small rockets, burns too violently, and too quickly in those that are large, because the fire is bigger, and the matter also driven closer together: hence it is that no gun-powder is used in the larger sort. In making up this composition, according to the different sizes of rockets, the following proportions must be observed.

For Rockets from 60 to 100 pounds, you must to three pounds of Saltpetre, add one pound of Sulphur, and two pounds of good Wood-coal.

If they are from 30 to 50 pounds, to 30 pounds of Saltpetre, put seven pounds of Sulphur, and sixteen pounds of Coal.

Rockets from 18 to 20 pounds, to twenty-one pounds of Saltpetre, require six of Sulphur and thirteen of Coal.

From

From 12 to 15 pounds, require to four pound of Saltpetre one pound of Sulphur, and two pounds of Coal.

If they be from 9 to 12 pounds; to sixty-two pounds of Saltpetre, add 9 pounds of Sulphur, and twenty of Coal.

From 6 to 9 pounds; add to seven pounds of Saltpetre, one of Sulphur, and two of Coal.

From 4 to 5 pounds; to eight pounds of Saltpetre, add one pound of Sulphur, and two of Coal.

From 2 to 3 pounds; to sixty pounds of Saltpetre, add two of Sulphur, and 15 of Coal.

For one pound; to sixteen pounds of Gunpowder, add one pound of Sulphur, and three of Coal; or to nine pounds of Powder, four of Saltpetre, one of Sulphur, and two of Coal.

For twelve ounces; put to nine pounds of Powder, four of Saltpetre, one of Sulphur, and two of Coal.

For eight ounces; add to thirty pounds of Powder, twenty-four of Saltpetre, three of Sulphur, and eight of Coal.

For five and six ounces; to thirty pounds of Powder, add twenty-four pounds of Saltpetre, three pounds of Sulphur, and eight pounds of Coal.

For four ounces; add to twenty-four pounds of Powder, four pounds of Saltpetre, two pounds of Sulphur, and three pounds of Coal.

For two and three ounces; to twenty-four pounds of Powder, put four pounds of Saltpetre, one pound of Sulphur, and three pounds of Coal.

For an half ounce, and an ounce; take fifteen pounds of Powder and two pounds of Coal.

For the smaller Rockets: to nine or ten pounds of Powder, add one pound, or one and a half of Coal.

Here follow also other proportions, which experience hath taught to succeed extremely well.

For Rockets that contain one or two ounces of matter: Add to one pound of Gun-powder, two ounces of good Coal; or, to one pound of Musket-powder, take one pound of coarse Cannon-powder; or, to nine ounces of Musket-powder, put two ounces of Coal: or to one ounce of Powder, an ounce and a half of Saltpetre, with as much Coal.

For Rockets of two or three ounces; add to four ounces of Powder, one ounce of Coal; or to nine ounces of Powder, two ounces of Saltpetre.

For a Rocket of four ounces; add to four pounds of Powder, one pound of Saltpetre, and four ounces of Coal, and if you please, half an ounce of Sulphur: or to one pound two ounces and a half of Powder, four ounces of Sulphur, and two ounces of Coal: or to one pound of Powder, four ounces of Saltpetre, and one ounce of Coal; or to seven ounces of Powder, four ounces of Saltpetre, and as much Coal: or, add to three ounces and an half of Powder, ten ounces of Saltpetre, and three ounces and an half of Coal. The composition will

will be yet more strong, if it be made up of ten ounces of Powder, three ounces and an half of Saltpetre, and three ounces of Coal.

For Rockets of five or six ounces, take two pounds five ounces of Powder, to half a pound of Saltpetre, two ounces of Sulphur, six ounces of Coal, and two ounces of Filings of Iron.

For Rockets of seven or eight ounces; add to seventeen ounces of Powder, four ounces of Saltpetre, and three ounces of Sulphur.

For Rockets from eight to ten ounces; to two pounds five ounces of Powder, put half a pound of Saltpetre, two ounces of Sulphur, seven ounces of Coal, and three ounces of Filings.

For Rockets from ten to twelve ounces; take to seventeen ounces of Powder, four ounces of Saltpetre, three ounces and an half of Sulphur, and one ounce of Coal.

For Rockets from fourteen to fifteen ounces, to two pounds four ounces of Powder must be added; nine ounces of Saltpetre, three ounces of Sulphur, five ounces of Coal, and three ounces of File dust.

For Rockets of one pound, to one pound of Powder, take one ounce of Sulphur, and three ounces of Coal.

For a Rocket of two pounds, add to one pound four ounces of Powder, twelve ounces of Saltpetre, one ounce of Sulphur, three ounces of Coal, and two ounces of File dust of Iron.

For

For a Rocket of three pounds, to thirty ounces of Saltpetre, put seven ounces and an half of Sulphur, and eleven ounces of Coal.

For Rockets of four, five, six, or seven pounds, add to thirty one pounds of Saltpetre, four pounds and an half of Sulphur, and ten pounds of Coal.

For Rockets of eight, nine, or ten pounds, take to eight pounds of Saltpetre, one pound four ounces of Sulphur, and two pounds twelve ounces of Coal.

The proportion of the different materials being thus determined, each of them must be well beaten, and searced apart, and afterwards weighed and mixed. Thus is your composition ready where-withal to charge your Coffins, which must be made of strong paper well pasted.

S P O R T XLVII.

To make a Rocket.

YOUR Coffins and different Compositions being in readiness, you must chuse a composition suitable to the largeness of your designed Rocket, which must neither be too wet nor too dry, but a little moistened with some oily liquor, or with brandy; then take your Coffin, the length of which must be proportioned to the bigness of its concavity; put it, with the Rammer, into the Mould; then put into it some of your composition, taking good care not to put in too much at a time, but only

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one

one spoonful or two ; then put in your Rammer, and with a Mallet suited to the bigness of the Coffin, strike three or four smart blows directly upon it ; then withdraw the Rammer again, and pour in an equal quantity of your composition, and drive it down in like manner with your Rammer and Mallet, giving the same number of blows ; continue thus doing till the coffin is filled to the height of the mould, or rather a little below it, that five or six folds of the paper may be doubled down upon the composition thus driven into the Coffin, which sometimes instead of paper is made of wood.

The Coffin being filled with the mixture, and the paper doubled down upon it, you must beat it hard with the Rammer and Mallet to press down the folds of the paper, upon which you may put some Corn-powder, that it may give a report. In this paper folded down, you must make three or four holes with a bodkin, which must penetrate to the composition, to set fire to the Stars, Serpents, and Ground-rockets, when such there are ; otherwise it will suffice to make one hole only, with a broach or bodkin, which must be neither too small nor too great, but about one fourth of the diameter of the bore, as straight as possible, and in the very middle, in order to fire the Corn powder.

S P O R T XLVIII.

*To make Sky-Rockets that mount into the Air,
with Sticks.*

IT is to be noted, that the head of a Rocket, is the highest end, by which it is loaded, and which rises first when it is fired: the neck of the Rocket, or its tail, is the lower end, where it was choaked or straitened, and the priming is put, which must be of good Corn-powder.

Your Rocket being charged, as was taught in the preceding Sport, you must have a long rod or stick, of some light wood, such as Osier, or Fir, which must be bigger and flat at one end, growing slenderer towards the other. This stick must be straight and smooth, without knots, and plained if need be. Its length and weight must be proportioned to the size of the Rocket, being six, seven, or eight times the length of it; to the larger end of this, where it is flattened, you must tie your Rocket, its head reaching a little beyond the end of the stick, and being thus fixed, lay it upon your finger two or three inches from the neck of the Rocket, which should then be exactly balanced by the stick, if it is rightly fitted; after which you have nothing to do, but to hang it loosely upon two nails, perpendicular to the horizon, with its head up, and then it is ready for firing. But if you would have it to rise very high, and in a straight line, you must put a pointed paper cap
K 2
upon

upon its head, and it will pierce the air with greater facility.

To these Rockets, for the greater diversion of the spectators, several other things may be added: as Petards or Crackers, thus; get a box of iron soldered, of a convenient bigness, fill it with fine grain-powder; put it into the Coffin upon the composition, with the touch-hole down, double the rest of the paper upon it to hold it fast till the mixture is consumed, and then firing it will give a report in the air.

You may add to them likewise, Stars, Golden-rain, Serpents, Fire-links, and other such agreeable works, the making of which shall be taught afterwards. In order to this, you must have in readiness an empty Coffin, of a larger diameter than your Rocket. This must be choaked at one end, so as only to admit the head of the Rocket, to which it must be fastened. Into this large Coffin, having first strewed the bottom of it with Meal-powder, you must put your Serpents, or Golden-rain, or Fire-links, with the primed end downwards; and amongst, and over your stars you must throw a little powder. Then you may cover this additional Coffin with a piece of paper, and fit to it a pointed cap as before, to facilitate its ascension.

S P O R T XLIX.

To make Sky-Rockets which rise into the air without a stick.

SKY-ROCKETS without sticks must be small, because they are held in the hand
from

from whence they rise, after you have put fire to the priming they are made as the foregoing; but that they may the better fly into the air, you must fit to them four wings disposed cross-wise, like the feathers of darts or arrows; their length must be one-third part of that of the rocket, their breadth at the lower part half their length, and their thickness about a sixth or eight part of the diameter of the orifice of the rocket.

Instead of four of these wings, you may use three of the same dimensions with equal success; but with this caution, that in placing them upon your rocket, the lower ends of them must be let down below the tail of it the length of one diameter of its orifice. There are many other ways of making these rockets, according to the various fancies of artists, which would be too tedious for this work.

If the composition for your Rockets is defective, as is known when they rise, either not at all, or with difficulty, or fall down again before consumption of the mixture; or when they mount not with an equal and upright motion, but turning and winding, or whirling in the air; to amend your composition, you must diminish the quantity of coal when it is too weak, and add to it if too strong, as it is when it bursts the rocket, the coal serving to abate the force of the powder, and to give a fine train to your rocket. Wherefore it would be convenient, before you make up a quantity of rockets, to try your mixture and correct its faults.

To preserve your rockets in good condition, they must be kept in a place neither too dry, nor too moist, but temperate; and the composition should not be made up, but upon occasion to use it. Your rocket must not be pierced till you design to play it; which must not be in a season of wind or rain, or when the nights are moist with fogs and mists, all which are prejudicial to the agreeable effects of a rocket.

If you would have your rocket to burn with a pale white flame, mix some Camphire with your composition; instead of which if you take raspings of Ivory, the flame will be of a clear silver-colour, but somewhat inclining to that of lead; if Colophony or Grecian pitch, it will be of a reddish copper-colour; if black, or common pitch, the flame will be dark and gloomy; if Sulphur, it will be blue; if Sal-armoniack, it will appear greenish; if crude Antimony, or the raspings of yellow Amber, it will emit flames of a like colour.

S P O R T L.

To make Ground Rockets, which run upon the earth.

ROCKETS that run along the Ground, called therefore, Ground-Rockets, require not so strong a composition, as those that mount into the air; and therefore continue longer burning as well as moving more slowly: Wherefore they vary from the others, as well in

in the dimensions of their Coffins, as in the composition wherewith these are charged. The length of the bore or concavity, may be eleven times that of its diameter; the rowler on which the coffin is made, may be five lines in diameter, and the rammer a little less, that it may go easily into the coffin without spoiling it.

The composition may be of Cannon-powder only, well beaten and searced till it is as fine as flower, wherewith you must fill the coffin, by little and little as before, within a finger's breadth of the brim of the mould; then doubling down one-third part of the paper, knock it down with the rammer and mallet, and after with a bodkin, make a small hole which may penetrate to the composition; then put in a pistol-charge of fine powder, doubling down some more of the paper upon it, the rest of which must be choak'd tying it hard with pack-thread.

These rockets being small, are charged only with powder finely pulverised, without any coal, herein differing from the large ones, that have no powder at all, except in their priming which in both sorts must be of well grained powder: The reason of which is, because in a greater concavity there is a greater fire acting upon a greater quantity of matter, and consequently with more violence; there being also a greater quantity of air to be rarified in a great than in a small rocket.

When you choak or streighten the end of your Rocket, whether small or great, you must

have a hook or staple driven into a post, or into a wall, to this tie one end of your cord, which must be of a size proportionable to your rocket, or to the bar of a window, and the other to a strong stick, which you must put between your legs: thus the cord being winded about your rocket in the designed place, you may draw, turning and straightening it by degrees as you desire.

S P O R T L I.

To make Rockets that fly on a line, called Air-Rockets.

THIS is done with ordinary rockets that must not be too big, by fastening to them two iron rings, or, which in my opinion is better, a wooden pipe or cane, through which must pass a well stretched line: thus if you set fire to your rocket, it will run along the line without ceasing till all the matter is spent.

If you would have your rocket to run back, as well as forward, after you have filled one half of the coffin with the composition, separate this from the empty half by a wheel of wood fitted exactly to the cavity; in the middle of this wheel must be a hole, from which a small pipe, filled with meal-powder, must pass along the middle of the empty half, which then must be filled with the composition; and so after the first half of the rocket is consumed, the fire being communicated by the little pipe, will light it

it at the other extremity, and so drive it back to the place from whence it came.

The same thing may be effected by the means of two rockets tied together, the tail of the one to the head of the other, one of which being burnt to the end fires the other, making it to run back: but lest the second should catch fire at the head, it must be defended with a cover of paper or waxed cloth.

This sort of rockets is commonly used to set fire to other machines in fire works for diversion, to which, for the greater pleasure, they give the figures of several animals, such as serpents or dragons, which then are called Flying Dragons; and are extremely agreeable, chiefly when filled with several other works, as golden rain, hairs dipt in wild fire, small nut-shells filled with the rocket composition, and many other diverting things, of which afterwards.

S P O R T LII.

To make Rockets that burn in the water, called Water-Rockets.

THOUGH the fire and water are opposite elements mutually destroying one another, yet the rockets we have hitherto described, being once lighted will continue to burn even in the water, and will have their full effect; but for as much as it is done under water, we are deprived of the pleasure, of beholding it. In order therefore, to make them to swim upon the

the water, we must alter somewhat the proportions of their mould, as well as the materials of their composition.

The mould, then, required to such rockets, may be eight inches in length, and its bore an inch over. The rowler must be of nine lines diameter, and the rammer not quite so thick: no needle is required to this mould.

The composition, if you would have your rocket burn on the water, with a clear flame like a candle, must be made of three ounces of powder beaten and seared, one pound of saltpetre, and eight ounces of sulphur mixed together: when you desire your rocket to appear on the water with a fine tail, you must, to eight ounces of common powder, add one pound of saltpetre, eight ounces of sulphur, and two ounces of coal.

The composition being prepared, and the coffin charged with it, as is taught above, put a fire-link at the end of it; and covering your rocket with wax, pitch, or rosin, to preserve the paper from the water, fasten to it a stick of white willow about two foot long, which will cause it to swim upon the water.

Many other different ways may such rockets be made without altering either the mould or composition, for which the curious may consult the authors that have writ particular treatises of Pyrotechny.

A rocket also may be made, which, after burning some time in the water, will throw up into the air sparkles and stars; which is done by

by dividing the rocket into two parts with a wooden wheel having a hole in the middle, one partition being filled with the common composition, the other with stars, having some powder strewed amongst them.

Moreover you may contrive a rocket, which having burnt one half of its time in the water, will mount up in the air with great swiftness; thus: having filled two equal coffins with good composition, past them together slightly only at the middle, the head of the one answering the tail of the other; betwixt them must pass a little pipe at the extremity, to light the other when one is consumed. Then fasten the rocket to which the other is joined, to a stick of such length and bigness as is required for balancing it, and to the lower end of the rocket tie a pack-thread, to which you must fasten a large musket-ball that must hang upon the stick by means of a bent wire. This done, set fire to your rocket, it being in the water: and its composition being consumed, will light, by means of a little pipe, the other rocket, which will mount into the air, through the strength of the fire, the first being kept down by the weight it sustains.

SPORT

S P O R T L I I I .

To make Fire Links.

A *Fire-Link*, so called from its resemblance, to the links of a saucidge, is a kind of Rocket, that is usually tied to the end of a bigger one, to render the effect more agreeable. I said, usually, because there are some of them made that fly into the air as Sky-rockets, and are called Flying Fire-links, to distinguish them from the others which are named fixed Fire-links. We shall here briefly teach the making of both sorts.

And first the fixed kind to be fastened to a Rocket is made thus: Take a coffin of what bigness you think fit, and having choaked it at the end, fill it with fine powder, and choak it at the other end: then roll it strongly with small cord from one end to the other, gluing the cord with good glue, to keep it fast, and to strengthen the coffin, that it may give the greater noise when it breaks: thus is your fire-link ready to be fastened to the end of a Rocket, either with paper, parchment, or cord, or otherwise; but note, that you must pierce the end of your Fire-link, which joins to the Rocket, and prime it with Corn-powder.

To make flying Fire links, you must have such coffins as for the former, only they must be a little longer, and having choaked them at one end, charge them with Corn powder, adding at least meal-powder to the thickness of
one

one inch, driving all down, as in Sky-rockets, with a mallet. Then strengthen the coffin with a line, as in the former, after you have choaked the other end, leaving a hole about the bigness of a goose quill, to which you must put a little moistened powder for priming.

Or, having choaked at one end, and charged your coffin, within one inch of the other end, choak it there, leaving only a small hole, which if quite shut up, or too small, must be opened with a bodkin; then fill up your empty space with powder finely flowered, or with the composition for Sky-rockets, which must be driven close with a rammer and mallet, doubling down the remaining paper, if any, upon your composition, which will give a fine tail to your link; and when you have made a hole in the middle of this last paper, and primed it, your flying link is ready to be thrown into the air, which is done thus:

You must provide guns or cannons with a vent at bottom, where there must be a tail somewhat long, which must pass through a piece of wood that it may reach to a fire conveyance running along underneath, to set fire to the cannons one after another, which will also throw up into the air the links with a noise in the same order.

S P O R T LIV.

To make Fire-Lances.

LANCES of fire, are long and thick pipes or cannons of wood, with handles at the end,

end, whereby they are made fast to stakes or posts, well fixed, that may sustain the force of the fire, having several holes to contain rockets or petards. They are used in festival fireworks that represent nocturnal fights, as well for throwing rockets, as making volleys of reports.

You must use them thus: Put a rocket into every hole, and fill the bore of the cannon with composition, which fired will, as it consumes, fire the rockets one after another, and throw them up into the air. But if you would have many thrown up at once, cover the bottom of the lance with composition, and thereupon place a long small pipe filled with the same composition, about which put your rockets, till you have filled your cannon, the primed end being downwards, that so firing the composition in the pipe, this may light that at the bottom of the lance, which firing the rockets, they will mount all at once into the air.

There may be many other ways of contriving Fire-Lances in imitation of this, of which I shall not speak. I shall only mention one other sort of these lances. This consists of a coffin made of strong paper well glued, which may be of what dimensions you think fit, according as it is designed to give more or less light; this must be filled with the Star-composition, pulverised and primed with meal-powder moistened: the lower end must be stopped with a round piece of wood, which must appear two inches

inches without the coffin, that thereby it may be fastened at pleasure.

Remark. The name of fiery or burning lances and pikes, is also given to a kind of pikes like a javelin or dart, with a strong iron-pointed head, called by the Latins *Phalorica*, and *Dardi di Fuoco* by the Italians, which were formerly thrown, being first fired against the enemies, either by the hand or from engines, being covered between the iron and wood with tow dipt in sulphur, resin, Jews' pitch, and boiling oil; where they lighted they stuck, setting on fire whatever was inflammable.

This sort of lances is not now in use, but instead of them we have Burning Arrows, that are not less terrible, though not much now in esteem; however, we will here gratify the curious with a brief description of them. Flaming Arrows are artificial firebrands thrown amongst the enemies' works, to reduce them to ashes: they are made thus——Prepare a little bag of strong coarse cloth, about the bigness of a goose's or swan's egg, of a globular or sphaeroidal figure, which must be filled with a composition made of four pounds of beaten powder, as much refined saltpetre, two pounds of sulphur, and one pound of Grecian pitch: or you may make it of two pounds of meal powder, eight pounds of saltpetre refined, two pounds of sulphur, one pound of camphire, and one pound of Colophony: or yet more simply thus——of three pounds of powder, four pounds of saltpetre, and two pounds of sulphur. With one of these

these mixtures fill the bag, pressing it hard, and make an hole through the middle of it lengthwise, to receive an arrow, like those of the ordinary bows or cross bows, the head of it remaining without the bag, which must be fastened so as it may not move or slide towards the feathers. This done; roll your bag with strong packthread as thick as possible from one end to another, and then cover it all over with meal-powder mixed with melted pitch. Thus it is ready to be shot out of a bow or cross-bow, after it is fired by two little holes made for that purpose near the head of your arrow.

S P O R T L V.

To make Stars for Sky-Rockets.

STARS are little balls about the bigness of a musquet-ball, or an hazle-nut, made of an inflammable composition, which gives a splendid light, resembling that of stars, from whence is the name. When they are put into the rocket, they must be covered with prepared tow, the manner of making which shall be taught, after that of stars.

They are made thus: to one pound of powder finely flowered, add four pounds of saltpetre, and two pounds of sulphur; and having mixed all very well, roll up about the bigness of a nutmeg of this mixture in a piece of old linen or in paper; then tie it well with packthread, and make a hole through the middle with a pretty big

big bodkin, to receive some prepared tow, which will serve for priming: this being lighted, fires the composition, which emitting a flame through both holes, gives the resemblance of a pretty large star.

If instead of a dry composition, you use a moist one in form of paste, you need only roll it into a little ball, without wrapping it up in any thing, save, if you will, in prepared tow, because of itself it will preserve its spherical figure; nor needs there any priming, because while moist you may roll it in meal-powder, which will stick to it, and when fired will light the composition, and this at falling forms itself into drops.

Remark. There are many other ways of making stars, too long now to be mentioned; I shall only here shew how to make Stars of Report, that is, stars that give a crack like that of a pistol or musquet, as follows:

Take small links, made as is taught in Sport 53, which you may choose either to roll with line or not, tie to one end of them, which must be pierced, your stars if made after the first manner, that is, of the dry composition: otherwise you need only leave a little piece of the coffin empty beyond the choak of the pierced end, to be filled with moist composition, having first primed your vent with grain powder.

You may also contrive stars, which, upon consumption of the composition, may appear to be turned into serpents, a thing easy to be performed.

formed by such as understand what precedes; upon which account, and because they are but little in use, I shall say no more of them.

S P O R T L V I .

To make prepared Tow for priming to Fire-works

PREPARED Tow, called also Pyrotechnical match, and Quick-match, to distinguish it from common match, is used for priming all sorts of machines for Fire-works of diversion, such as rockets, fire-lances, stars, and the like; and it is made as follows:

Take thread of flax, hemp or cotton, and double it eight or nine times, if it is for priming your large rockets, or fiery lances; but four or five times only, if it is to be put thro' your stars. Having made it of a bigness proportioned to your designed use, and twisted it, but not too hard, wet it in clean water, which must be after squeezed out with your hands. Then put some gun-powder in a little water, so as to thicken it a little; in this soak your match well, turning and stirring it till it is thoroughly impregnated with the powder; and then taking it out, roll it in some good powder dust, and hang it upon lines to dry either in the sun or shade: thus you have a Pyrotechnical match ready for use on all occasions.

Common Match, called also Fire cord, is thus made: Take an unglazed earthen pot, cover its bottom with red sand well washed and dried;

dried; upon this lay spiral-wise plain match of cotton, or well cleaned tow, half an inch thick the distance of half an inch being between each revolution, and then cover it with sand; upon which again place a lay of match as before, and upon this another of sand, and so interchangeably till the pot is full, but finishing always with a lay of sand: then cover it with an earthen cover, and lute with clay the joining, so as no air may get entrance. This done put burning coals round the pot, and after it has been kept hot for some hours, let it cool of itself; so your match is prepared, which will burn without smoak or offensive smell.

S P O R T LVII.

To make Golden-rain for Sky-rockets.

TH E R E are some Sky-rockets, which in falling make little waves in the air, like unto hair half curled, and are therefore called Hairy-rockets; they end in a sort of rain of fire, called Golden-rain.—'Tis thus made.

Fill with the composition for sky-rockets goose quills, the feathers being cut off; putting some wet powder in the open end of each, both to keep in the composition, and to serve for priming: with these fill the head of your sky-rocket, and it will end in a golden rain, very agreeable to behold.

Remark. This golden rain calls to my mind a Pyrotechical-hail, so called from its resemblance

blance to the natural, which is a quantity of small hard bodies, being either pieces of flint, round stones, leaden bullets, or square pieces of iron, inclosed in a cartridge of wood, iron, or copper, and is therefore called cartridge or case-shot; they are used in war, either in open field to disorder an enemy's army, or in a siege to drive them away from a breach or gate to be seized, being shot either out of a mortar, or a great-gun of a large bore.

S P O R T L V I I I .

To represent with Rockets, several figures in the air.

IF you take a rocket of the larger sort, and place round the head of it many small ones, fixing their sticks all round the large coffin upon the head of your big rocket, which uses to contain the head works, ordering it so, that your small rockets take fire, whilst the great one is mounting up, you will have the resemblance of a tree, very delightful to the sight; whereof the big one will represent the trunk, and the little ones the branches.

But if the small rockets take fire when the great one is half turned in the air, they will have the appearance of a comet: and when the large one is altogether turned, so that its head points downwards to the earth, they will exhibit the similitude of a fountain of fire.

If

If you put on the head of a large rocket many goose-quills, the feathers being cut off, filled with sky-rocket composition, as in the preceding sport; when fired, they will appear to those under them as a fine shower of fire; but to those who view them on one side, like half curled hair, very delightful to the view.

Finally, with serpents tied to a rocket with packthread by the ends which are not fired, leaving two or three inches of the thread between each, you may represent at pleasure several sorts of figures most entertaining and agreeable to the sight.

S P O R T LIX.

To make Fire-pots for Fire-works of diversion.

A Pot of fire is a large Coffin filled with Rockets that take fire all together, and are discharged from the pot without hurting it. The bottom of the pot must be covered with powder-dust, which being fired by a match that must pass through a hole in the middle of the pot, will set fire to all the rockets at once.

When there are many Fire-pots, they must be covered with single paper, that they may not play all at once; otherways, one when fired might set fire to another; and you must use only a single leaf of paper, that it may not hinder the rockets to fly out. Pots of fire are also made for war-service.

S P O R T L X.

To make Shining-balls for diversion, and for service in war.

FIRST, to make shining-balls for recreation; to four pounds of saltpetre, put six pounds of sulphur, two pounds of crude antimony, four pounds of colophony, and four pounds of coal: or, to two pounds of saltpetre, take one pound of sulphur, as much antimony, two pounds of colophony, as much coal, and one pound of black pitch; melt these, being well beaten, in a kettle, or in a glazed earthen pot, and thereinto throw such a quantity of hards of flax or of hemp, as will just suffice to imbibe all the liquor, of which as it cools make little pellets or round balls, to be covered over with prepared tow, which I taught to make in Sport 56, and after put into sky-rockets, or balls for diversion, as is usual to be done with fiery stars.

Next, to make shining or flaming balls for service in war, to be thrown from a mortar, against the enemy, you must melt in a kettle, or glazed earthen pot, as above, equal parts of sulphur, black pitch, rosin and turpentine, into which dip an iron, or stone bullet, somewhat lower than the bore of the mortar, and when its surface is covered with this matter, roll it in corn-powder: which done, cover it over with callico, and dip it again into the same liquor; rolling it after in grain-powder; this must be reiterated several times, covering, dipping and rowling

rolling it, till it fills exactly the bore of the mortar or cannon, into which you design to put it, remembering still to end your operations with rolling it in grain-powder, that being put into your piece, immediately above the charge of the powder, it may take fire as it is thrown into the air against the enemy, either to annoy them, or to discover their designs, which is usually done in sieges.

Instead of these shining balls, red-hot-balls are more frequently used for offending the enemy, by burning them, their houses or works. These bullets are of iron, and being heated red hot in a furnace are thus used. Your cannon being charged with powder, freed from corns, and pointed something upwards, you must have in readiness a cylinder of wood fitted exactly to its bore, which you must put into your gun next the powder, and upon it you must ram down a wad of wet straw, hay, tow or hemp, or some such moist materials; then putting in your red-hot ball with a ladle, immediately put fire to your gun.

S P O R T LXI.

To make a Wheel of Fire-works.

A Wheel of Fire, or Fire-works, is a wheel of light wood, set round with rockets of a mid-size, the head of one regarding the tail of another, that when the first is spent, it may set fire to the next, which makes the wheel turn

round its fixed axle-tree without intermission, till all the rockets are consumed.

Upon this account it is called a Fire-wheel, and it is also called a *Fiery Sun*, because placed horizontally upon a stake somewhat large and perpendicular to the horizon, it turns round, and represents a sun in night combats, which is very diverting.

You may also make fire-wheels which have a situation perpendicular to the horizon, and turn upon an axis parallel to it, very agreeable to behold. Fire-wheels are likewise used to light other works at a distance, in ascending or descending upon a stretched rope, like flying dragons; and on many other occasions, to the pleasure of the spectators.

S P O R T LXII.

To make a Balloon, or fiery Foot-ball.

BALLOONS are coffins of a large diameter shot out of a mortar whither one pleases, filled commonly with Serpents about the thickness of a Ground-rocket, but not so long, with two small fire-links of the same length and breadth, which being fired by their priming, burst the coffin, this having below a fire-conveyance, at the mouth of which there is a priming of cotton dipt in powder.

The coffin is made with a thick wooden roller, about which is rolled strong card-paper glued, to keep it from undoing, which being
choaked

choaked below, a hole is made there for a fire-conveyance, filled with a composition more slow than that of Ground-rockets, being like to that of Sky-rockets: after this it may be filled with Serpents, and sometimes with Stars, and then choaked above.

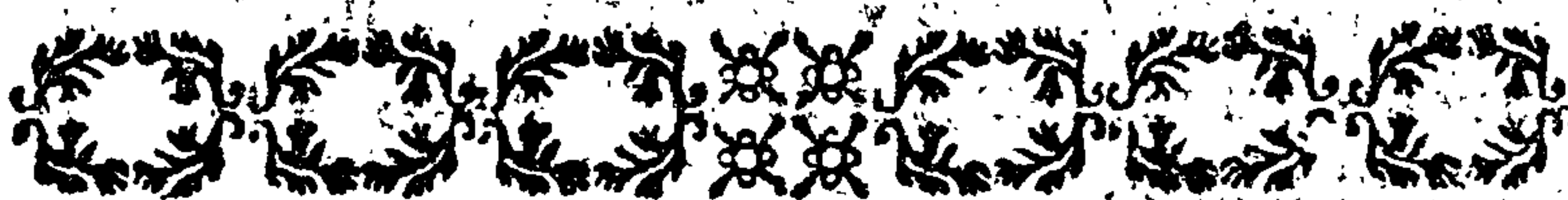
S P O R T LXIII.

To make an Ointment excellent for curing all sorts of burnings.

BOIL, over a gentle fire, in common water Hogs' lard, or the fat of fresh pork, skimming it perpetually till no further scum arises; then expose it thus melted to cool in the clear open air three or four nights. After this melt the same lard or grease in an earthen vessel over a slow fire, and strain it through a linen cloth into cold water, and after wash it well in fair river or fountain water, to take away its salt, which will make it become white as snow. Finally, being thus purified, put it up in a glazed earthen vessel, to be kept for use upon occasion.

If it falls out, as commonly it happens, that by a burning, blisters rise upon the skin, they must not be cut or broken, till the ointment has been used to it for three or four days. You may also use the following, which you will find to be of great efficacy, and is made of Hogs'-lard melted and mixed with two drachms of the water of Night shade, and one drachm of oil of Saturn;

Saturn ; or with two ounces of juice of Onions, and one ounce of oil of Walnuts.



P A R T I I I .

Miscellaneous S P O R T S .

S P O R T L X I V .

To describe an artificial Lantern, by which one may read at night at a great distance.

MA K E a lantern in the form of a cylinder or of a small cask laid on one side ; put in one of its two ends a concave parabolic glass, in order to apply to its focus the flame of a wax candle, the light of which will reflect to a great distance in passing through the other end that ought to be open and will appear with such a splendor, that by it one may read at night very small letters at a great distance, with telescopes ; and those who see the light of the candle at a great distance, will take it to be a great fire, which will be still the lighter if the lantern is tinned within, and made in the form of an oblique.

The

The Magical Lantern.

We likewise make use of such a glass for a magical lantern, so called, because by means of it we can make any thing appear on the white wall of a dark room; such as monsters and fearful apparitions, which the ignorant impute to magic. The light reflected by virtue of this glass passes through a hole in the lantern, in which there is a lens of glass; and between them there is a thin piece of wood containing several glasses painted with monstrous and formidable figures, which they move up and down through a slit in the body of the lantern, and which cast their representation to any opposite wall in the same colours and proportions, but much enlarged.

S P O R T LXV.

By the means of two plain Looking-glasses to make a Face appear under different forms.

HAVING placed one of the two glasses horizontally, raise the other to about right angle, over the first; and while the two glasses continue in this posture, if you come up to the perpendicular glass, you will see your face quite deformed and imperfect; for it will appear without forehead, eyes, nose or ears, and nothing will be seen but a mouth and a chin raised bold. Do but incline the glass never so little from the perpendicular, and your face will appear with all its parts excepting the eyes and the forehead.

forehead. Stoop it a little more, and you will see two noses and four eyes; and then a little further, and you will see three noses and six eyes. Continue to incline it still a little more, and you will see nothing but two noses, two mouths and two chins; and then a little further again and you will see one nose, and one mouth. At last incline a little further, that is, till the angle of inclination comes to be forty-four degrees, and your face will quite disappear.

If you incline the two glasses the one towards the other, you will see your faces perfect and intire; and by the different inclinations, you will see the representation of your face, upright and inverted alternately, &c.

S P O R T LXVI.

To describe a vertical dial upon a pane of glass, so as to denote the hours without a gnomon.

I Once made such a dial for a friend after the following manner:

I took off a pane of glass that was soldered on the out-side to the frame of a window, and calculating the thickness of the frame for the gnomon, had the pane glewed on again to the inside of the frame, allotting to the meridian line a situation perpendicular to the horizon, as it should be in vertical dials, and on the out-side, I caused to be glewed to the frame opposite to the dial, a strong piece of paper unsoiled, that
so

so the rays of the sun might penetrate it the less, and keep the surface of the dial darker. Then to distinguish the hours without a style, I made a little hole in the paper with a pin, over against the foot of the style marked upon the dial; and thus the hole representing the tip or end of the style, and the rays of the sun passing through it, cast upon the glass a small light, that pointed out the hours very prettily in the obscurity of the dial.

S P O R T LXVII.

To find in all parts and at all times, the four cardinal points of the world, without seeing the sun, or the stars, or making use of a compass.

THE four cardinal parts of the world, viz. the east, the west, the south and the north, are easily found by a compass, the needle of which being touched with a loadstone, turns always one of its points towards the south and the other towards the north, which is enough to direct us to east and west; for when one sets his face to the north, the east is on his right, and the west is on his left hand.

The north is easily distinguished in the night by the stars, particularly by minding the polar star, which is but two degrees distant from the arctic pole; and in the day-time astronomers mark the meridian line upon an horizontal plain, by means of the two points of a shadow marked

marked before and after noon upon the circumference of a circle described from the point of the stylus, the shadow of which is made use of to shew by its extremity upon that circumference two points equally remote from the meridian.

But without all these helps you may at all times, and in all parts, find out the meridian line, after the following manner.

Take a platter or basin full of water, and when the water is settled and still, put softly into it an iron or steel needle, such as a common sewing needle; and if the needle is dry, and be laid all along upon the surface of the water, it will not sink; but after several turns it will stop in the plan of the meridian circle, so that it represents the meridian line; and by consequence one end of it will point to the south, and the other to the north: but without seeing the sun or the stars, it is not easy to know which of the two ends points to the south and which to the north.

Father Kircher lays down an easy way of knowing south and north. He orders you to cut horizontally a very straight tree growing in the middle of a plain at a distance from any eminence or wall that may shelter it from the wind or the rays of the sun: in the section of that trunk you will find several curve lines round the sap which lie closer on one side than the other.

And, as he says, the north lies on that side where the lines are most contracted, perhaps because the cold arising from the north binds up,

up, and the heat from the south spreads and rarifies the humours and matter, of which these crooked lines are formed. These lines, says that author, are as the circumferences of concentric circles in ebony or Brazil wood.

S P O R T LXVIII.

To break with a stick another stick resting upon two glasses without breaking the glasses.

THE stick that is to be broken must not be very thick, nor yet lean much upon the glasses; it ought, as near as possible, to be equally thick all over, for the easier finding of its center of gravity, which will then be in the middle.

The stick being thus qualified, we lay its two ends, which ought to terminate in a point, upon the brim or edge of two glasses of equal height, so that the stick does not lean to one side or end more than the other, and the two points ends rest but lightly upon the edge of each glass, to the end that when it bends a little thro' the violence of the stroke, it may easily slip off, and break at the same time. This done, we take another stick, and with that give a smart blow upon the midling point, which being the centre of gravity will receive all the force of the blow; thus will the stick break, and that the more easily that the blow is violent, and fall clear of the two glasses which remain unbroken, because the stick lay but very gently and equally

ally upon the brim of each ; for if it rests more upon one glass than the other, it will press that one most, and so may break it.

S P O R T LXIX.

To make a lamp fit to carry in one's pocket, that shall not go out though you roll it upon the ground.

TO make a lamp that never spills its oil, and never goes out in any position whatsoever, make fast the vessel that contains the oil and the match, to an iron or brass ring, with two small pivots or hinges diametrically opposite, that so the vessel may by its weight continue in æquilibrium round the two hinges and turn with freedom within the circle, so as to keep always to an horizontal position, as in your sea compasses, which have two such circles to keep them horizontally : and in like manner this first circle ought to have two other pivots diametrically opposite, which enter into another circle of the same substance ; and that second circle has two other little hinges inserted in another concave body that surrounds the whole lamp. Thus the lamp with its two circles may turn freely upon its six hinges, which give to the lamp when it is turned, six different positions, viz. up and down, forwards and backwards, to the right and left, and which serve to keep the lamp in an horizontal position, which, being in the middle does always rest up-
on

on its centre of gravity, that is, its center of gravity is always in the line of direction which hinders the oil to spill, turn it which way you will.

S P O R T LXX.

To take up a boat that is sunk with a cargo of goods.

IF a boat sinks in a deep river, you may bring her up again, by getting two other boats, one empty, and the other deep loaded with some heavy substance, as stones, &c. You must tie these two boats to the boat that is sunk with two ropes, and extending the rope of the deep laden boat, unload her into the other that is empty; which will raise the first boat a little, and make it draw along with it the boat that is under water, and at the same time make the second boat swim so much deeper in the water. The second boat being thus loaded, you must bend her rope and unload her again into the empty boat, and thereupon she becoming lighter, will rise and draw the boat under water so far further up. Thus you continue to load and unload till you bring the boat even with the water, and then row her to the side.

S P O R T LXXI.

To make a boat go of itself up a rapid current.

TH E more rapid a river is, the easier it is to make a boat go of itself up against the current, by a rope and a wheel with its axel-tree that has wings like the wings or sweeps of a mill-wheel.

Fix the wheel with its axel-tree at the place to which you would have the boat conducted, and let its sweeps be as deep in the water as there is occasion for turning it round: tie a rope to the boat, and to the axel-tree of the wheel, which turning with its axel-tree by virtue of the rapidity of the water, will wind up the rope on its axel-tree, and so by the successive abbreviation of the rope, drag it against the current to the place proposed: which it will reach so much the sooner that the current is rapid, the rapidity quickening the motion of the wheel.

S P O R T LXXII.

To know which of two different waters is the lightest, without any scales.

TA K E a solid body, the specific gravity of which is less than that of water, deal or fir-wood, for instance; and put it into each of the two waters, and rest assured that it will sink deeper in the lighter than in the heavier water; and so by observing the difference of the sinking you

you will know which is the lightest water, and consequently the wholesomest for drinking.

S P O R T LXXIII.

To know if a suspicious piece of Money is good or bad.

IF it be a piece of silver that is not very thick, as a crown or half crown, the goodness of which you want to try; take another piece of good silver of equal ballance with it, and tie both pieces with thread or horse-hair to the scales of an exact ballance (to avoid the wetting of the scales themselves) and dip the two pieces thus tied in water; for then if they are of equal goodness, that is, of equal purity, they will hang in equilibrium in the water as well as in the air; but if the piece in question is lighter in the water than in the other, it is certainly false, that is, there is some other metal mixed with it that has less specific gravity than silver, such as copper; if it is heavier than the other, it is likewise bad, as being mixed with a metal of greater specific gravity than silver, such as lead.

If the piece proposed is very thick, such as that crown of gold that Hiero king of Syracuse sent to Archimedes to know if the goldsmith had put into it all the 18 pounds of gold that he had given him for that end; take a piece of pure gold of equal weight with the crown proposed, viz. 18 pounds, and without taking the trouble of weighing them in water, put them in-

to a vessel full of water, one after another, and that which drives out most water, must necessarily be mixed with another metal of less specific gravity than gold, as taking up more space though of equal weight.

S P O R T LXXIV.

To know how the wind stands, without stirring out of one's chamber.

FIX to the cieling of your room a circle divided into 32 equal parts, with the names of the 32 rumps or wind-points, the points of north and south being upon the meridian line. The circles or dial thus divided, must have a needle or hand moveable round its centre, like the hand of a watch or clock; and that hand must be fixed to an axeltree that is perpendicular to the horizon, and may move easily upon the least wind, by virtue of a fane on its upper end above the roof of the house, and then the wind turning the fane, will at the same time turn its axeltree, and the hand that is fixed to it, which will accordingly point to the rump from whence the wind blows.

S P O R T LXXV.

When two vessels or chests are like one another, and of equal weight, being filled with different metals, to distinguish the one from the other.

THIS Sport is easily performed, if we consider that two pieces of different metals of equal

equal weight in air, do not weigh equally in water; because that of greatest specific gravity takes up a lesser space in water, it being a certain truth, that any metal weighs less in water than in air, by reason of the water, the room of which it fills. For example, if the water weighs a pound, the metal will weigh in that water a pound less than in the air. This gravitation diminishes more or less according as the specific gravity of the metal is greater than that of the water.

We will suppose then two chests perfectly like one another, of equal weight in the air, one of which is full of gold, and the other of silver; we weigh them in water, and that which then weighs down the other must needs be the gold chest, the specific gravity of gold being greater than that of silver, which makes the gold lose less of its gravitation in water than the silver. We know by experience, that gold loses in water about an eighteenth part only, whereas silver loses near a tenth part: so that if each of the two chests weighs in the air, for example, 180 pounds, the chest that is full of gold will lose in the water ten pounds of its weight; and the chest that is full of silver will lose eighteen; that is, the chest full of gold will weigh 170 pounds, and that of silver only 162.

Or, if you will, consider that gold is of a greater specific gravity than silver, the chest full of gold, though similar and of equal weight with the other, must needs have a lesser bulk

than the other. And therefore if you dip separately each of them into a vessel full of water, you may conclude that the chest which expels less water has the lesser bulk, and consequently contains the gold.

S P O R T LXXVI.

To measure the depth of the sea.

TIE a great weight to a very long cord, or rope; and let it fall into the sea till you find it can descend no farther, which will happen when the weight touches the bottom of the sea, if the quantity or bulk of water the room of which is taken up by the weight and the rope, weighs less than the weight and rope themselves; for if they weighed more, the weight would cease to descend, though it did not touch the bottom of the sea.

Thus one may be deceived in measuring the length of a rope let down into the water, in order to determine the depth of the sea; and therefore to prevent mistakes, you had best tie to the end of the same rope, another weight heavier than the former, and if this weight does not sink the rope deeper than the other did, you may rest assured that the length of the rope is the true depth of the sea: if it does sink the rope deeper, you must tie a third weight yet heavier, and so on till you find two weights of unequal gravitation that run just the same length of the rope, upon which you may conclude that the
length

length of the wet rope is certainly the same with the depth of the sea.

S P O R T LXXVII.

To represent lightning in a room.

TH E room in which you are to represent lightning must not be large, but quite dark, and so very close that the air cannot readily enter it. The room being thus in order, take a bason into it with spirit of wine and camphyr, which must boil there till it is all consumed, and nothing left in the bason. This will rarify the camphyr, and turn it into a very subtle vapour, which will disperse itself all over the room; insomuch that if any one enters the room with a lighted flambeau, all the imprisoned vapour will in a moment take fire, and appear as lightning, but without hurting either the room or the spectators.

Remark. Camphyr is of a nature so proper to retain and keep an unextinguishable fire, that it will burn entirely, and that very easily upon ice or among snow, which it melts, notwithstanding their coldness; and if it be reduced to powder and thrown upon the surface of any still water, and then lighted, it will produce a very pleasant sort of fire, for the water will appear all fire and flame; the reason of which I take to be, because the camphyr is of a fat nature which resists water, and of a light and fiery substance, which the fire grasps so keenly, that it is impossible

fible for this substance to disengage itself when once it is entangled.

S P O R T LXXVIII.

To melt at the flame of a lamp a ball of lead in paper, without burning the paper.

TAKE a very round and smooth leaden ball, wrap it up in white paper, that is not rumpled, but clings equally about the ball without wrinkles, at least as far as is possible; hold the ball thus wrapped up over the flame of a lamp or flambeau, and it will grow hot by degrees, and in a little time melt and fall down in drops through a hole in the paper without burning it.

S P O R T LXXIX.

To represent an Iris or rainbow in a room.

EVERY one knows that the rainbow is a great arch of a circle that appears all on a sudden in the clouds before or after the rain, towards that part of the air that is opposite to the sun, by virtue of the resolution of the cloud into rain; this arch is adorned with several different colours, of which the principal are five in number, namely red which is outermost, yellow, green, blue, and violet and purple, which is interior.

This Iris seldom appears alone, and is called the first and the principal rainbow, to distinguish

it from another that commonly appears along with it, and for that reason is called the second rainbow, the colours of which are not so lively as those of the first, though they are disposed after the same manner, but in a contrary order, upon which account a great many take it for a reflection of the first.

If you want to represent, at one time two such Iris's in your room, put water into your mouth and step to the window (upon which the sun is supposed to shine) then turn your back to the sun, and your face to the dark part of the room; and blow the water which is in your mouth, making it spurt out with violence into little drops or atoms; and among these little atoms or vapours, you will see by the rays of the sun, two rainbows resembling the two that appear in the heavens in rainy weather.

Oftentimes we see rainbows in water-works or spouts, when we stand between the sun and the fountain, especially when the wind blows hard, for then it disperses and divides the water into little drops. Which is full evidence, that the rainbow, which the philosophers admire as much as the ignorant people do thunder, is formed by the reflection and refraction of the rays of the sun darted against several little drops of water, that fall from the clouds in time of rain.

A rainbow may likewise be very easily represented, in a room with a window that the sun shines upon, by a triangular prism exposed to the rays of the sun, which in passing through the

the glass, will by their different reflections and refractions produce upon the wall or ceiling of the room, a very agreeable Iris, or at least a texture of several different colours resembling those of the rainbow; and the further the ceiling or wall is distant, and the more it is dark, the colours will appear the more charming and lively. You may likewise imitate the colours of the rainbow by exposing to the sun a sphere of chrystal or glass, or a glass full of clean water.

S P O R T LXXX.

To make a consort of musick of several parts with only one voice.

THE sound conveyed distinctly to the ear, by remote bodies, against which the air is driven by the voice of an animal or otherwise, and then reflected, is what we call an echo; which is sometimes double, triple, &c. when the voice is strong enough to make several bodies, at different distances, beat back at several times the parts of the air to our ears, so that one echo is not sooner ended than another begins.

Though most echoes make us hear only the last words of the voice, because the air, though strongly impressed, has not the same force at the end that it had at the beginning; yet it may be so contrived as to make a consort of music of several parts, that is, a consort of several songs tuned together, by only one voice

or

or one instrument, to the sound of which the eccho answers.

For if the eccho answers only once to the voice or the sound of the instrument, he who sings or plays may make a Duo, that is, a music of two parts; and again a Trio or music of three parts, if the eccho answers twice. But indeed he must be an expert musician, and one that is well versed in varying the tune and the note.

Thus commencing, for example at Ut, he may begin Sol a little before the eccho answers, so as to finish the pronounciation of Sol by the time that the eccho has compleated its answer, and then he will have a fifth, which is a perfect consonance in music; and in like manner, if at the same time with the eccho's answering to the second note Sol, or a little before, he repeats it upon a higher or lower note, he will make a diapason or eighth, which is perfect harmony in music. And so on, if he has a mind to continue the chace with the eccho, and sing alone the two parts.

To this purpose we see by experience in several churches, when they are singing, that there seems to be many more parts in the chorus than there really are, the quantity of eccho's making the air to resound on all sides, and so multiplying the voice and redoubling the chorus.

To make a string of a vial shake without touching it.

CHOOSE at pleasure three strings in a viol, or any other instrument of that sort without any intermediating string, and tune the first and the third to the same note, without touching that in the middle; then strike one of the two strings thus tuned pretty hard with a bow, and you will find that when it shakes the other will tremble sensibly and visibly, and the middle string, though nearer, shall stir no manner of way.

This Sport may likewise be resolved by two stringed instruments of the same sort, as two viols, two lutes, two harps, two spinets, &c. by putting the two in the same tune, and then placing them at a convenient distance, and in a proper position; for one of the two instruments being touched with a middling force, will move the other, that is, the strings of the other, which are supposed to be in unison, will produce such another harmony, especially if the strings in one and the other instrument are equally long and equally thick. For this I can assign no other reason but experience.

S P O R T LXXXII.

To make a deaf man bear the sound of a musical instrument.

IT must be a stringed instrument, with a neck of some length, as a lute, a guitarre, or the like; and before you begin to play, you must by signs direct the deaf man to take hold with his teeth of the end of the neck of the instrument; for then if one strikes the strings with a bow one after another, the sound will enter the deaf man's mouth, and be conveyed to the organ of hearing through the hole in the palate: and thus the deaf man will hear with a great deal of pleasure the sound of the instrument, as has been several times experienced. Nay, those who are not deaf may make the experiment upon themselves, by stopping their ears so as not to hear the instrument, and then holding the end of the instrument in their teeth while another touches the strings.

S P O R T LXXXIII.

To make an egg enter a vial without breaking.

LET the neck of the vial be never so strait, an egg will go into it without breaking, if it be first steeped in very strong vinegar, for in process of time the vinegar does so soften it, that the shell will bend and extend lengthways without breaking. And when it is in cold water thrown upon it will recover its primitive hardness, and, as Cardan says, its primitive figure.

S P O R T

S P O R T LXXXIV.

To make an egg mount up of itself.

M A K E a little hole in the shell of the egg, and so take out the yolk and the white, and fill the egg-shell with dew, then stop up the hole and expose it to the rays of the sun at noon-day; for then the dew not being able to bear the light, nor too great heat, will rise up with the egg shell, especially if it leans against a little stick or piece of wood, that slopes never so little, and if the hole is well stopped. May dew is said to be best; and it is observed by the farmers, that the more May abounds in dew, the more plentifully does the earth bring forth: for dew being a subtle vapour produced in the morning by a weak heat, and preserved by a moderate cold, it is very well disposed for the reception of celestial virtues; and when it insinuates itself into vegetables, it communicates to them the virtue it retains; and hence it comes that plants moistened with it, thrive better than when they are nourished with spring, well, or river water.

S P O R T LXXXV.

To make water freeze any time in a hot room.

F I L L a vial with warm water, the neck of which is somewhat narrow, and having stopped it close, put it in a vessel full of snow, mixed

mixed with common salt and saltpetre, so as to leave the vial covered all over with snow: and in a little time the water will be quite frozen, though in the summer time, and in a very hot room.

If you throw cold water with snow upon a table, and upon the snow set a platter full of snow with a sufficient quantity of salt and saltpetre pounded; the salt and the saltpetre will make the snow so cold, that in a little time the water under the platter will be turned to ice, and make the platter stick so fast to the table, that you cannot move it without some difficulty.

Remark. The saltpetre and sal-armoniac are likewise possessed of the virtue of making water so extremely cold, that if you put a sufficient quantity of them in common water, it will become so cold that your teeth can scarce bear it. They might therefore be very usefully employed in summer for cooling wine or any other liquor, by setting the wine bottles in water thus refrigerated.

If you dissolve a pound of nitre in a pail of water, the water will be excessive cold, and so very proper for the uses above-mentioned. It is well known that wine is cooled with ice; and in regard ice cannot always be had in summer, I shall here prescribe a way of making it.

To make ice in summer.

To make ice in summer, put two ounces of refined saltpetre, and half an ounce of Florentine Oris, into an earthen bottle filled with boiling

ing water, stop the bottle close, and convey it forthwith into a very deep well, and there let it steep in the well-water for two or three hours, at the end of which you will find the water in the bottle all ice: so you have nothing to do but to break your bottle and take out your ice.

S P O R T LXXXVI.

To kindle a fire by the sun-beams.

THIS Sport may be performed either by refraction in using lenticular glasses thicker in the middle than in the sides, called burning-glasses, through which when the rays pass they refract and unite into one point called the Focus, at which you may light a match or any other combustible matter: or else by reflexion, in using a concave looking-glass of metal well polished in its concavity, which may be either spherical or parabolic, and is likewise called a burning glass, but much better than the former sort; for by it you may in a moment set fire to a piece of wood, and in a short time melt lead, and even iron, and vitrify stone.

S P O R T LXXXVII.

To make a fowl roasting at the fire, turn round of itself with the spit.

TAKE a wren and spit it upon a hazel stick, and lay it down before the fire, the

two ends of the hazle stick being supported by something that is firm; and you will see with admiration the spit and the bird turn by little and little without discontinuing, till it is quite roasted. This experiment was first found out by Cardinal Palotti at *Rome*, who shewed it to father Kircher, in order to know the physical cause of it; which to my mind is easily discovered; for the hazle wood is composed of several long and porous fibres, into which the heat insinuates itself, and so makes it turn round when the wood is hung right.

S P O R T LXXXVIII.

To make an egg stand on its smallest end, without falling, upon a smooth plain such as glass.

PL.A.C.E. a looking-glass quite level, or horizontally, without inclining to either side, toss the egg with your hand till the yolk bursts, and the matter of it is equally dispersed through all the parts of the white, so that the white and the yolk make but one body. Then set the end of the egg upon the horizontal plain, holding it till it is upright, and then it will continue in that situation without falling, by reason of the æquilibrium made on all sides by the parts of the yolk equally mixed with the white, so that the center of gravity in the egg continues in the line of direction.

S P O R T LXXIX.

To make a piece of gold or silver disappear, without altering the position of the eye or the piece, or the intervention of any thing.

PUT the piece of gold in a porringer full of water, or a vessel that is broader than it is deep, and let the eye be in such a position, as just barely to see the piece at the bottom over the brim of the vessel; then take out the water, and though the porringer continues in the same position as well as the eye, the piece which appeared before by virtue of the refraction made in the water, will then be covered from the sight by the sides of the porringer.

S P O R T XC.

To make a Loaf dance while it is baking in the Oven.

PUT into the dough a nut-shell filled with live sulphur, saltpetre and quicksilver, and stopped close; as soon as the heat comes to it, the bread will dance in the oven; which is occasioned by the nature of quicksilver, for it can bear no heat without being in a continual motion. Thus, by the means of quicksilver put into a pot where pease are to be boiled, all the pease will leap out of the pot as soon as the water begins to heat. In like manner quicksilver put into hot bread, will make it dance up and down the table.

SPORT

S P O R T XCI.

To see in a dark room what passes abroad.

MAKE your room so close and dark, that the light can come in no where but through a little hole left in a window upon which the sun shines; over against this hole, at a reasonable distance from it, place some white paper, or a piece of linen; and you will see every thing that passes by the outside of the window appear on the paper or linen, only their figures are inverted.

S P O R T XCII.

To hold a glass full of water, with the mouth down, so as that the water shall not run out.

TAKE a glass full of water, cover it with a cup that is a little hollow, inverting the cup upon the glass; hold the cup firm in this position with one hand, and the glass with the other, then with a jerk turn the glass and the cup upside down, and so the cup will stand upright, and the glass will be inverted, resting its mouth upon the interior bottom of the cup. This done, you will find that part of the water contained in the glass will run out by the void space between the bottom of the cup and the brim of the glass; and when that space is filled, so that the water in it reaches the brim of the glass, all passage being then denied to the air,

so that it cannot enter the glass, nor succeed in the room of the water, the water remaining in the glass will not fall lower, but continue suspended in the glass.

If you would have a little more water descend into the cup, you must with a pipe or otherwise draw the water out of the cup, to give passage to the air in the glass; upon which part of the water will fall into the glass till it has stopped up the passage of the air afresh, in which case no more will come down; or, without seeking out the water in the cup, you may incline the cup and glass so that the water in the cup shall quit one side of the brim of the glass, and so give passage to the air, which will then suffer the water in the glass to descend till the passage is stopped again.

This Sport may likewise be performed by covering the brim of the glass that is full of water, with a leaf of strong paper, and then turn the glass, as above; and without holding your hand any longer upon the paper, you will find it as it were glewed for some time to the brim of the glass, and during that time the water will be kept in the glass.

S P O R T X C I I I .

To make new Wine keep its sweetness for several Years.

MR. LENTIN informs us, that if you let new wine heat of itself, it loses in a little time

time all its sweetness, especially if the casks are left open; but if you boil it upon a fire immediately after the grapes are pressed, most of the volatile principles of the sweetness concentrate, and link themselves with the more fixed parts of the wine, which preserves its sweetness for several years.

Remark. A sweet and new wine may preserve its sweetness at least a whole year, if you pitch the cask well both within and on the outside, to hinder the water to penetrate into it, and so spoil the wine, which ought to be put into it before it boils; and keep the cask well stopped in a cistern of water, so as to be covered all over for a month or thirty days; and then take out the cask and place it in a cellar.

In the year 1692, I had a cask full of Burgundy wine brought me in the summer to *Paris* by water, which immediately upon its arrival was clapped into my cellar, and after a few days standing, I found it boiling as if it had been quite new, and that it had reassumed its former sweetness, which continued about a month, and after that it proved extraordinary good wine. Some tell you that a piece of cheese or pumice-stone thrown into the cask, will break the violence of fermenting wine.

To recover the sweetness of new wine.

When the new wine has lost its sweetness, it may be recovered by casking it up immediately, and putting in the bottom of the cask half a pound of mustard-seed, less or more, according to the size of the cask.

S P O R T XCIV.

To know when there is water in wine, and to separate it from the wine.

IF the wine is neither sweet nor new, but fine and clear of its lee, you may know, (according to Porta and father Schott) whether it is mixed with water or not, by throwing into it apples or pears, for if the wine is unmixed they will sink to the bottom, if it is mixed they will swim above, because the specific gravity of water is greater than that of wine.

Some order wild apples or pears, and if these cannot be had, ripe apples or pears. Others make use of an egg, and alledge, that when the wine is pure the eggs falls swiftly to the bottom, but if it is mixed with water, the egg descends more slowly, the water having, by virtue of its gravity more force to bear up the egg than the wine has.

Now the contrary will happen, if the wine be sweet and new, that is, when such wine is unmixed, the egg will descend slower than when it is mixed; by reason that new wine unmixed is by virtue of its lee heavier than water, and consequently becomes lighter, by the addition of water.

When you have discovered that the wine is mixed, you may separate the water by a dry bulrush, according to Mizauld; for the rush being a plant that grows and thrives in watery marshy places, if it be dried, and one end of it

put

put into mixed wine, the water will insinuate itself into the rush, and so the wine will be left alone. By the same reason, the rush may serve to discover whether the wine is mixed with water or not.

Remark. On the other hand, some pretend you may separate the wine from the water, by putting in a long narrow piece of linen, woollen or cotton cloth, one end of which hangs out of the vessel, as if the wine being lighter would rise and flow out upon the cloth while the water stays behind; but this and several other ways for the same purpose, are disapproved by other authors.

To pour water into wine without mixing.

You may pour wine upon water, without mixing, if you put a toast of bread upon the water in a glass, and while this toast swims above the water, pour in the wine very softly; for then you will see the water remain unmixed at the bottom of the glass without any alteration in its colour.

To know if milk is mixed with water.

Here by the bye I shall shew you the way of knowing when water is mixed with milk; put a little stick into the milk, then pull it out, and let a drop of the milk fall from it upon the nail of your thumb; and if the milk is pure, the drop being thick will stand for some time upon your nail; but if it is diluted with water it will run off immediately.

To turn water seemingly into wine.

You may turn water into wine in appearance, by setting a vial full of water in a cask full of wine, turning the mouth of the vial downwards; for then the water will run out, and the vial will be filled with wine; which the ignorant will take to be a turning water into wine.

S P O R T XCV.

To make a metallic body swim above water.

THOUGH the specific gravity of water is inferior to that of metals, and consequently water is incapable, absolutely speaking, to bear up a metallic body, such as a ball of lead, yet this ball may be flatted and beat out to a very thin plate, which when very dry and put softly upon still water, will swim upon it without sinking, by virtue of its dryness. Thus we see a steel needle will swim upon the water, when it is dry, and laid softly lengthways upon the surface of still water.

But if you would have a metallic body to swim necessarily upon water, you must reduce it to a very thin plate, and that concave like a kettle, in which case the air it contains weighs less than the water, whose room it possesses. It is by this contrivance that copper boats or pontons are made for passing whole armies over rivers without any danger.

Remark. If you put this concave metallic vessel upon the water, with its mouth perpendicularly

cularly down, it will still swim, by reason that the air contained in its cavity finds no exit; in-
somuch that if you push it under water and hold it there by force, the detained air will keep the bottom from being wet on the inside. And by the same reason you may have a burning coal in the bottom, and find it not extinguished when you take it out of the water, provided you do not hold it long under water, for fire stands in need of air to keep it in.

S P O R T XCVI.

To make Aquafortis put up close in a bottle boil without fire.

PUT a small quantity of Aquafortis, and of the filings of brass in a bottle, and you will see so great an ebullition, that the bottle will appear quite full, and be so hot that you cannot touch it without burning yourself.

Remark. In like manner if you mix oil of tartar and oil of vitriol together, you will presently see a very great ebullition with a sensible heat, though neither of these liquors is composed of any combustible matter.

Of Aquafortis and Aqua Regia.

Aquafortis is so called with respect to its strength, in dissolving almost all metals and minerals. It is commonly a distillation from saltpetre and vitriol or green copperas; and it is yet better, if it be distillation from saltpetre
and

and roach-allum. It dissolves all metals but gold; but is rendered capable of the dissolution of gold by dissolving sal-armoniack or sea-salt in it, after which it assumes the name of Aqua Regia.

Of Sal-Armoniac, Roach Allum, and Saltpetre.

To avoid all obscurity of terms; I shall here acquaint you by the bye, that Sal-armoniack is a composition of bay-salt, chimney-foot, and the urine of animals: That roach-allum is a mineral earthy sharp salt, filled with an acid spirit, which is oftentimes found condensed in the veins of the earth, or is taken from aluminous springs by evaporation; or is found among mineral stones, and disengaged from them by dissolution in water and evaporation. And in fine, that saltpetre is a salt that is partly sulphureous and volatile, and partly terrestrial; it is found in the dark cavernous places of the earth, and likewise in stables, by reason of the great quantity of volatile-salt in the urine and excrements of animals, which joins in with the salt of the earth by the continual action of the air.

Vitriol.

The oil of Vitriol (mentioned above) is a caustic oil distilled by a strong reverberating fire from Vitriol. Now, Vitriol is a mineral salt, approaching to the nature of roach-allum, which is found chrystallised in the earth of such mines as abound in metals, which gives us to know that it contains in it some metallic substance, and

and particularly iron or copper. When it is loaded with copper, if you rub it against iron, it will stain it with a copper colour. But it is best for all manner of preparations when it partakes most of iron.

Tartar.

The oil of Tartar (mentioned above), is distilled from Tartar along with the spirit, from which it is separated by a funnel lined with brown paper. Tartar itself is an earthy incorruptible substance, formed like a reddish crust round the inside of wine-casks, which thickens and congeals to the hardness of a stone, and is separated from the pure parts of the wine, by the action of the fermentative spirit.

S P O R T XCVII.

To make fulminating or thundering powder.

T A K E three parts of saltpetre, two parts of salt of tartar, and one part of sulphur pounded and mixed together; heat in a spoon sixty grains of this composition, and it will fly away with a fearful noise like thunder, and as loud as a cannon, breaking through the spoon and every thing underneath it, for it exerts itself downwards, contrary to the nature of gun-powder, which exerts itself upwards.

Salt of Tartar.

The salt of tartar, here used, is only a solution in water of the black substance that remains after

after the distillation of the oil of tartar, and an evaporation of that solution to a dry salt, which must be kept very close, least the moisture of the air should melt it.

S P O R T XCVIII.

To make the Aurum Fulminans, or thundering gold.

PUT into a matraass, upon hot sand, the filings of fine gold, with a tripple quantity of Aqua Regia, which will dissolve the gold: mix this solution with a sextuple quantity of spring water, and then pour upon it, drop by drop, the oil of tartar or volatile spirit of sal-armoniac, till the ebuillition ceases, and the corrosion of the Aqua-regia is over, for then the powder will precipitate to the bottom, which may be dulcified with warm water, and dried with a very slow fire.

This powder is much stronger than the last described; for if you set fire to 20 grains of it, it will act with more violence, and have a louder crack than half a pound of gun-powder, and two grains of it kindled at a candle have a stronger report than a musket-shot.

S P O R T XCIX.

To make the Sympathetic-Powder.

THE Sympathetic-powder is nothing else but the Roman vitriol calcined and reduced

ced to a white light powder, which is said to cure wounds at a distance, by being put upon a linen cloth dipped in the wounded person's blood, or upon a sword whereon is the blood or pus that comes out of the wound. This cloth or sword is wrapped up in a white linen cloth, which is opened every day, in order to strew some fresh powder upon the blood or pus of the wound. This course they continue till the wound is perfectly cured, which happens the sooner, if the cloth upon which is the blood and the powder, is kept in a place that is neither too hot nor too cold, nor too moist. Nay, it is necessary sometimes to shift the cloth from place to place, according to the different dispositions of the wound, by putting it, for example, in a cold place, when the patient finds an excessive heat in the wound.

To calcine the vitriol for the sympathetic powder, take some Roman vitriol, when the sun is in the sign of Leo, or in the month of July, dissolve it in rain water, and filtrate the water through sinking pater. Then let the water evaporate upon a gentle fire, and you will find at the bottom the vitriol in little hard stones of a fire-green colour. Spread these stones carefully, and expose them to the rays of the sun, stirring them often (with a wooden spatula; not an iron spatula, because the spirits of the vitriol are ready to join in with iron, which would rob the sympathetic powder of its volatile spirits, in which all its virtue consists) that the stones may be the better penetrated by the sun, and calcined
and

and reduced to a powder, which will be as white as snow. And to render the substance of the vitriol more pure and homogenous, the dissolution, filtration, coagulation and calcination ought to be repeated three times.

This wonderful powder must be carefully kept in a vial close stopped, and in a dry place, for the least moisture of the air may turn it to vitriol again, and so make it lose its sympathetic virtue.

We are told that this powder stops all bleedings, and mitigates very much all sorts of pains in any part of the body, particularly the tooth-ach; and that, by application not to the part affected, but to the blood taken from it, and covered up in a linen cloth as above.

Remark. The chymists have another calcination of vitriol called Colcothar, which being put into the nose stops a bleeding at the nose, and provokes to sneeze; being of sovereign use in rouzing the senses, wherefore it is given in lethargies. It is also successfully used for drying up wounds and ulcers. This colcothar is only the vitriol kept melted upon a fire till all its humidity is evaporated, and it is reduced to a hard reddish-brown mass, whereby it is rendered fit for the cure of the aforesaid maladies, and many others not here mentioned.

S P O R T C.

Of the magnetical cure of diseases by transplantation.

THE magnetic cure by transplantation, is, that which is performed by communicating the disease to some beast, tree, or herb; and, as some will have it, is founded upon the efflux of the morbidic particles, which pass by insensible transpiration out of the body of the patient into another animal or plant.

Froman informs us, that a young student got rid of a malignant fever by giving it to a dog that lay in the bed with him, and died of it: which if true, must needs proceeds from the insensible transpiration of the subtile matter, that thereupon entered the pores of the dog.

Thomas Bartholin says, his uncle was cured of a violent cholic, by applying a dog to his belly, which was thereupon seized with it; and that his maid-servant was cured of the tooth-ach by clapping the same dog to her cheek, and when the dog was gone from her he howled and made such motions, as gave them to know he had the maid's tooth-ach.

Hoffman speaks of a man cured of the gout by a dog lying in the bed with him, who thereupon was seized with it. And frequently after the dog had fits of the gout, as his master had used to have before. However this be, certain it is, that dogs are often subject to the gout, without

without any infection from men; and this, and the other stories of transplantation are not here offered for conclusive proofs, but by way of recreation.

Monfieur de Valemont, who seems inclinable to believe transplantation of diseases, says, it is done not only by insensible transpiration, but likewise by sweat, by urine, by the blood, by the hair, or by taking up what falls from the skin, upon a strong friction. For this he brings several instances, and particularly that which follows :

A person of quality in *England*, used to cure the jaundice at a great distance from the patient, by mixing the ashes of ash-wood with the patient's urine; and making of that composition three or seven, or nine little balls, with a hole in each of them, in which he put a leaf of saffron, and then filled it up with the same urine. This done, he hung these balls in a private place where no body could touch them; and from that time the disease began to abate.

The great virtues of the Ash-tree.

Remark. The Ash, which is a common tree all over Europe, has merited the appellation of the Vulnerary wood, by reason of its peculiar property in curing several diseases, and above all wounds and ulcers. Not to mention the almost incredible virtues ascribed to it, it is said to stop bleeding at the nose, if the face be but rubbed with the wood, and then rubbed with fair water, and if the patient holds in the hand
of

of that side where the bleeding is, a piece of the wood till it heats his hand.

S P O R T C I.

To stop a bleeding at the nose, or at any other part of the body.

FATHER Schott the Jesuit says, that to stop a bleeding at the nose, you need only to hold to the nose the dung of an ass very hot, wrapped up in an handkerchief, upon the plea that the smell will presently stop it ——— Wecher did the same with hogs dung, very hot done up in fine taffata, and put into the nose.

I have several times experienced, that a piece of red coral held in the mouth, will stop a bleeding at the nose. Some tell you that the constriction of the thumb of the side of the nostril that bleeds, will do the business.

To stop the bleeding of a wound, take a linen cloth in the spring when the frogs lay their eggs in the water, and wash it in that water till it is well impregnated with the frogs eggs; then dry it at the sun; and after repeating this impregnation and deficcation three or four times, keep the cloth to be applied to the wound twice in the form of a cataplasm. We are told the second application will do.

S P O R T C H.

To prepare an ointment that will cure a wound at a distance.

TH E ointment mentioned by Paracelsus is prepared thus, according to Goclenius: Take of the Usnea or moss of the scull of a man that was hanged, two ounces; mummy, human blood, of each half an ounce; earth-worms washed in water or wine, and dried, two ounces and a half; human fat, two ounces; the fat of a wild boar, and the fat of a bear, of each half an ounce; oil of linseed and oil of turpentine, of each two drachms.

John Baptist Porta prescribes it a little otherwise by throwing in some bole armoniac, and leaving out the earth-worms, and the bears and boars fat. But let the composition be which it will, it must be well mixed and beat in a mortar, and kept in a long narrow vial. Some say it should be made when the sun is in Libra.—The way of using it is this:

Put into the ointment the weapon or instrument that gave the wound, and leave it there; then let the patient wash his wound every morning with his own urine, and apply nothing else to it; after it is well washed and cleansed, let him tie it up tight with a clean white linen cloth, and he will find it will heal without any pain.

Monsieur Vallemont says, if you cannot get the instrument with which the wound was given, you

you may take another, which if gently conveyed into the wound, and impregnated with the blood and animal spirits residing there, will have the same effect. He adds, that if you want a speedy cure, you must anoint the instrument often, otherwise you may let it lie a day or two without touching it.

The effect of this unguent he imputes to the subtle particles, which are these little agents that disengage themselves from the most spirituous and transpirable ingredients of which this unguent is composed.

To add to the credibility of its operation, he quotes father Lana, who observed that when the vines in *France* were in flower, the wines in *Germany*, though at a great distance, suffer an effervescence; which he explained by the effluvia of subtle matter, making these to reach as far as the stars, and alledging that if the atoms, which transpire from the terrestrial globe, were not carried to the stars, and sent back from the stars to the earth by a perpetual flux and reflux, there would be no physical commerce between the heavens and the earth.

S P O R T CIII.

To pierce the head of a Pullet with a needle without killing it.

THIS is a very easy Sport, for there is a place in the middle of a pullet's head, that may be pierced without hurting the Cerebellum.

bellum. But the needle must not be kept in above a quarter of an hour.

S P O R T C I V .

To make handsome faces appear pale and hideous in a dark room.

BURN some brandy and common salt in a glass, then put out the fire and all the lights in the room; and the particles of the salt and brandy evaporating into the air shut up in the room, will make the faces of the people in the room appear through that air hideous and frightful.

I intimated above, that if, instead of brandy, you take good spirits of wine mixed with camphyr in a glazed earthen pan put upon hot burning coals, he that enters the room with a lighted candle will be agreeably surprised; for the candle setting fire to the particles of the spirit and the camphyr, with which the air is replenished, that air will seem to be all in a fire and the person will see himself in the midst of flames without being burned.

- S P O R T C V .

To lift a mortar of ten pounds weight with a wine-glass.

TURN the mortar up-side down, rub the bottom very smooth, so that there be not the

the least unevenness ; make a circle the bigness of the rim of the glass with clay used by distillers ; then take a lighted paper, put it into the glass, turn it down on the mortar, fixing it to the circle of clay and luting it quickly with the same, so as not the least air may enter : when the paper is burned out, and the glass is cool, proceed to the experiment and you will find it answer.

S P O R T C V I.

To make an egg move upon a table.

TAKE an egg, make a small hole at each end, and after you have blown out the contents through these holes, put a live leech into it, then stop up the holes with wax : the egg thus prepared being set upon the table, place some fresh water at a little distance, which the leech immediately smelling will make towards, and give a motion to the egg, to the agreeable surprize of the spectators.

S P O R T C V I I.

To make an artificial flame.

AN Y of the following oils, mixed with compound spirits of Nitre, burst into a flame, viz. oil of Carroways, Cloves, Sassafras, Guaiacum, Box, Camphyr, Pepper, Hartshorn, Blood, and many other oils.

S P O R T CVIII.

To make an artificial earthquake.

TO twenty pounds of iron filings add as many pounds of sulphur; mix, work and temper the whole together with a little water, so as to form a mass, half moist and half dry: this being buried three or four foot under ground, in six or seven hours time will have a prodigious effect; the earth will begin to tremble, crack and smoke, and fire and flame burst through.

S P O R T CIX.

To prepare a lamp that will make a company look of any colour you please.

IF you would have a green assembly, order a lamp to be made of green transparent glass, put green oil and a green wick in it, and the company will appear green. If you would have a black company, you must do the same in black and so in any colour.

S P O R T CX.

To bake an egg upon your head.

TAKE a loaf of bread just from the oven, cut a hole in it, put the egg in this hole and cover it with the bread cut out, then wrap the loaf in a linen cloth, and hold it upon your head

head for some time, the egg will be soon done enough.

S P O R T CXI.

To make flames issue from an egg.

BL O W out the contents of an egg and dry the shell in the sun, then fill it with brimstone, saltpetre, and unquenched lime; close the holes of the egg through which you blew the contents, then throw it into water and it will flame.

S P O R T CXII.

To make a ring leap about.

PREPARE a ring made of copper, hollow, fill it with quicksilver, and stop close the hole again that the quicksilver may not find any vent. This ring put upon an iron plate made very hot with coals underneath, will skip about briskly.

S P O R T CXIII.

Of a deceitful bowl to play withal.

MA K E a hole in one side of the bowl, and cast melted lead therein, and then stop up the hole close, that the knavery or deceit be not perceived; you will have pleasure to see that notwithstanding the bowl is cast directly to the play, how it will turn away sideways; for that on that part of the bowl which

is heavier upon one side than the other, it never will go truly right, if artificially it be not corrected, which will hazard the game to those who know it not; but if it be known that the leady side in rolling, be always under or above, it may go indifferently right; if otherwise, the weight will carry it always sideways.

S P O R T CXIV.

To contrive a vessel which keeps its liquor when filled to a certain height, but loses or spills it all when filled a little fuller with the same liquor.

TAKE a glass and run through the middle of it a small bended pipe or crane, open at the end next the bottom of the glass, and likewise at the other end, which must be lower than the bottom of the glass; for then the water or wine poured into the glass continues in it while the branch is filling, and till it comes to the bend or the uppermost part of the crane, which withal should be a little lower than the upper edge of the glass: but after that if you continue to pour more in, it will rise higher in the concavity of the glass, and not finding place for a farther ascent into the crane by reason of its bending downwards, it will change its ascent into a descent through the branch, and continue to descend and run out by the end, as long as you continue to pour in; nay, when you have

have done pouring, you will see that all that was in the glass before is gone.

You may make the water run out at the lower end, though the glass is not filled up to the top of the crane, namely, by sucking at the lower aperture the air contained in the crane, for then the water will necessarily succeed in the room of the air, and continue to descend through the branch, till the glass is empty, especially if the orifice touches the bottom of the glass.

Or else, run the small pipe perpendicular down through the glass; let the pipe be open at both ends, the uppermost of which ought to be a little lower than the brim of the glass, and the other end a little lower than the bottom of the glass. Put this small pipe in another larger pipe stopped at the upper end, which must be a little higher than the end of the first and smaller pipe, and open at the lower end, which must touch the bottom of the glass if you would have all the water run out, which it will do when it rises, for then passing through the orifice of the pipe, it will enter the pipe by one end and run out at the other.

S P O R T CXV.

To make a coach that a man may travel in without horses.

THE two four-wheels must be little, and moveable round their common Axeltree, as
in

in the ordinary coachess and the hinder wheel must be large, and firmly fixed to their common axeltree, insomuch that the axeltree cannot move, without the wheels move along with it.

Round the middle of the axeltree put a trundle-head, with strong and close spindles, and near to that fix upon the beam a notched wheel, the notches of which may catch the spindles of the trundle-head, and so in turning with the handle, that wheel round its axeltree, which ought to be perpendicular to the horizon, it will turn the trundle, and with that the axeltree and the wheels, which will thereupon set forward the coach, without horses or any other animal. I need not tell you that the axeltree must enter into the beam, in order to turn within it.

There was invented at Paris some years ago, a coach or chaise, which a footman behind it made to go with his two feet alternately, by virtue of two little wheels hid in the box between the two hind-wheels, and made fast to the axeltree of the coach.

In short, the contrivance of the machine is this: In a roller, the two ends of which are made fast to the box behind the chaise, is a pully upon which runs the rope that fastens the end of the planks, upon which the footman puts his feet, where is a piece of wood that keeps fast the two planks at the other end, allowing them to move up and down by the two ropes, tied to their two ends, where there are two little plates of iron which serve to turn the wheels

wheels that are fixed to their axeltree, which is likewise fixed to the two great wheels.

Thus, you will readily apprehend that the footman putting his feet alternately upon the end of each plank, one of the plates will turn out of the notched wheels; for example, if he leans with his foot upon one plank, it descends and raises the other, which can't rise but at the same time the plate of iron that enters the notches of the wheel, must needs make it turn with its axeltree, and consequently the two great wheels: Then the footman, leaning upon one plank, the weight of his body will make it descend and raise the other plank, which turns the wheel again; and so the motion will be continued.

'Tis easy to imagine that while the two hind-wheels advance, the two fore-wheels must likewise advance; and that these will always advance straight if the person that sits in the chaise manages them with reins made fast to the fore-beam.

S P O R T CXVI.

*To make two Images, one shall light a candle,
and the other blow it out.*

UPON the side of a wall make the figure of two images, in the mouth of each put a pipe or quill, so artificially that it be not perceived, in one of which place saltpetre very fine and dry and pulverised, and at the end set a little match of paper: in the other quill
sulphur

fulphur beaten small. Then, holding a lighted candle in your hand, say to one of those images by way of command, *Blow out the candle*, then lighting the paper with the candle, the saltpetre will blow out the candle immediately, and going to the other image (before the snuff of the candle be out) touch the sulphur with it, and say, *Light the candle*, and it will immediately be lighted.

S P O R T CXVII.

To seem to turn water into wine.

TAKE four beer-bowl-glasses, rub one in the inside with a piece of allum; let the second have a drop of vinegar in it, the third empty, and the fourth as much clean water in it as your mouth will contain: have ready in your mouth a clean rag with ground brasil tied close in it, that the bulk may be no bigger than a small nut, which must lie betwixt your hind teeth and your cheek; then take off the water out of the glass into your mouth, and return it into the glass that hath the drop of vinegar in it, which will cause it to have the perfect colour of sack; then turn it in your mouth again and chew your rag of brasil betwixt your teeth, and squirt the liquor into the empty glass, and it will have the perfect colour and smell of claret; returning the brasil into its former place, take the liquor into your mouth again, and presently squirt it into the glass you rubbed with allum, and

and it will have the perfect colour of mulberry-wine; and so in many other ways, which for brevity I omit.

S P O R T CXVIII.

To deceive one with seeming pieces of tobacco-pipe.

R OUL up a piece of white paper as hard as your lottery tickets, till it is as thick as a tobacco-pipe, then fasten the outward edge with a little starch or paste, having so done, cut the ends even; have this in your hand, break two pieces in the sight of the company, shake the three together in your hat, then cast them upon the table, saying, How many pieces of pipe is there under the hat? Every one will be apt to say, three; lift up the hat the better to urge them, clapping it down presently, saying, now I'll hold you a wager there are but two pieces of pipe under the hat; which when laid, take up the hat, and their folly will soon be discerned, by your cutting the paper with your knife.

S P O R T CXIX.

To write letters secretly, that cannot be discover'd

T A K E a sheet of white paper, and double it in the middle, then cut holes through both the half sheets, let the holes be cut like the panes of glass windows, or other forms
what

what you best fancy, and then with a pin prick two little holes at each end and cut your paper in two halves, give one half to your friend (to whom you intend to write) the other half keep to yourself: now when you do write, lay your cut paper on a half sheet of writing paper, and stick two pins through the two holes that it stir not; then through those holes that you did cut, write your mind to your friend; when you have done, take off your paper with the holes again, and then write some other idle words both before and after your lines; but if they were written to make some little sense, it would carry the less suspicion; then seal it up and send it.

When your friend hath received it, he must lay his paper on the same, putting pins into the pin holes, and then he can read nothing but your mind which you writ, for all the rest of the lines are covered.

S P O R T CXX. *Secret Writing.*

WRITE a letter (what you please) on one side of paper with common ink, then turn your paper, and write on the other side with milk, (that which you would have secret) and let it dry; but this must be written with a clean pen: Now when you would read it, hold that side which is written with ink to the fire, and the milky letters will then shew blueish on the other side, which may be perfectly discerned.

S P O R T CXXI. *Secret Writing.*

PUT the powder of roach allum into a little water, and with that write upon the paper: when the letters are dry they will disappear; but clap the paper in fair water, and the letters will look white and shining, the paper being a little blacked with the allum.

S P O R T CXXII. *Secret Writing.*

WRIT E with the water in which sal-ar-moniack well pulverised is dissolved: when the letters thus written are dry, they will disappear, but hold them near the fire, and then they become visible again. The same may be performed by writing with the juice of a lemon, or an onion.

S P O R T CXXIII.

To make Glass-Drops.

GLASS-DROPS are thick little pieces of glass, made almost like a drop which have a long slender end, which being broken at its extremity, the drop breaks presently with a crack, and flies into white powder and little fragments to two or three foot round.

These drops, which have excited the curiosity, and perplexed the reason of most philosophers, are made by letting a little of the melted matter of which the ordinary glasses are made, fall

fall into a vessel full of cold water; for then this melted matter which is very glutinous while 'tis red, makes a long string, by which they hold the drop in the middle of the water, where it cools and hardens in a little time; after which they separate the string which is out of the water, so that the remaining part in the water does not break, commonly called a glass-drop, to this drop there sticks a small end, part of which may be separated, by making it red at the flame of a candle, without breaking the drop; nor will ~~this~~ drop break if you lay it upon wood, and with a hammer strike upon its thickest part, for its external parts are very hard, and support one another like a vault. And they only break upon bending the slender end till it breaks, by virtue of the spring raised by that effort in all its parts, which shake and tremble like an extended string, put into motion by forcing it to bend; whence it comes that these parts do in a little time return with very great velocity to their first disposition; and that the parts which are less united, and only contiguous, as it were, disunite and separate, and that occasions the disunion and separation of all the rest, and their flying all about with a noise.— See upon this head Mr. MARIOTTE's discourse of the nature of the air published in 1679, in which he has, in my opinion, wrote more pertinently of this subject, than any one besides.

S P O R T

S P O R T CXXIV.

To represent the four Elements in a vial.

TH E four elements of which the author of nature has composed the elementary world are the *earth, water, air* and *fire*; of which, the earth being the heaviest, is said to have the lowermost station in the centre of the world: water being lighter covers the earth; air being lighter than water covers it; and at last fire the lightest of all surrounds the air. So that in this sense these four make four concentrical orbs, the common center of which is the center of the world.

We may represent the four elements in this order: in a long vial of glass or chrystal, by the help of four heterogeneous liquors, that is liquors of a different specific gravity, which are of such qualities, that, though shaken together by a violent agitation, they soon after return to their natural stations, and all the particles of one and the same liquor unite in a separate body from the rest, the lighter giving way to the heavier.

To represent the earth, make use of crude antimony, or blue smalt well refined, or black smalt coarsly pounded, which by its weight will sink to the bottom of the vial.

To represent water, pour upon the last the terrestrious substance of the spirit of tartar, or calcined tartar, or the clean solution of potashes, with a little roach-azur, which will give a sea colour.

P

To

To represent the air, pour upon this composition spirit of wine rectified three times, till it has a colour of air; or else the most spirituous brandy with a little turnsol, which will give it a celestial blew or air colour.

To represent fire, pour upon all three the oil of Belin, which by its colour, likeness and subtilty, will make a pretty near resemblance.

SPORT CXXV. *A Trick.*

TAKE a ball (or any thing else) in each hand, and stretching them as far as you can asunder, lay a wager with any person, that without bringing your hands nearer together, or throwing either ball from one hand to the other, you will have them both in which hand the person pleases who lays the wager with you: which is no more than to lay one ball down upon the table, and turning yourself round take it up with the other hand, without bringing your hands any closer.

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